V. N. Karazin Kharkiv National University
School of Mathematics and Computer Sciences

6-th International Conference “Differential Equations and Control Theory”

DECT 2023

Book of abstracts

V. N. Karazin
Kharkiv National University

Zachodniopomorski
Uniwersytet Technologiczny
w Szczecinie

October 11-13, 2023

Kharkiv, Ukraine

UDC 517

Book of abstracts of the 6-th International Conference "Differential Equations and control Theory" (DECT 2023).

The book contains abstracts of talks and of posters presented at the 6-th International Conference "Differential Equations and control Theory" (DECT 2023). The conference took place at V. N. Karazin Kharkiv National University on October 11-13, 2023.

For researchers in the field of differential equations and mathematical control theory.

The abstracts are given as submitted by the authors.

Keywords: ordinary and partial differential equations, control theory, linear and nonlinear control systems, systems with delay, controllability, stability, stabilizability, bounded and optimal control, boundary-value problem, approximation, mathematical modelling.

Approved for publication by the decision of the Scientific Council of the School of Mathematics and Computer Sciences of V. N. Karazin Kharkiv National University,
rep. No. 10 from October, 24, 2023.

Organizers:

School of Mathematics and Computer Sciences, V. N. Karazin Kharkiv National University (Ukraine),

Faculty of Computer Science and Information Technology and Faculty of Electrical Engineering, West Pomeranian University of Technology in Szczecin (Poland).

Address of the Organizing Committee:

Department of Applied Mathematics,
V. N. Karazin Kharkiv National University,
Svobody sqr. 4,
61022, Kharkiv, Ukraine
e-mail: appmath@karazin.ua

https://dect.univer.kharkov.ua

Contents

Homogeneous approximation of series of iterated integrals and time optimality

Daria Andreieva, *Kharkiv, Ukraine*
Svetlana Ignatovich, *Kharkiv, Ukraine*

We consider affine control systems with output of the form

$$\dot{x} = \sum_{i=1}^{m} X_i(x)u_i, \quad y = h(x), \tag{1}$$

where $X_1(x), \ldots, X_m(x)$ are real analytic vector fields and $h(x)$ is a real analytic scalar function defined in a neighborhood of the origin in $\mathbb{R}^n$. Let $x(t;u)$ denote the trajectory of the system starting at $x(0) = 0$ and corresponding to the control $u(t) = (u_1(t), \ldots, u_m(t))$. Then the output $y(t) = h(x(t;u))$ can be found in the form of a series of iterated integrals

$$y(t) = \sum_{I} c_I \eta_I(t,u), \tag{2}$$

where $I = (i_1, \ldots, i_k)$ are multi-indices, c_I are scalar coefficients, and $\eta_I(t,u)$ are iterated integrals. Such series can be studied within an algebraic approach since iterated integrals form a free associative algebra. The constraint $u_1^2(t) + \cdots + u_m^2(t) \le 1$ induces a gradation in this algebra. Namely, it is natural to define the *order* of the iterated integral $\eta_I(t,u)$ as $|I|$, i.e., the length of the multi-index. This allows us to introduce the concept of a homogeneous approximation of the series (2): we say that the sum of the terms of minimal order from the series (2) is *the homogeneous approximation of the initial series (2)*.

In the talk, we give an algebraic description of the introduced homogeneous approximation and discuss its connection with the time-optimal control problem for the system (1).

[1] Sklyar, G.M., Ignatovich, S.Yu.: Free algebras and noncommutative power series in the analysis of nonlinear control systems: an application to approximation problems. Dissertationes Math. (Rozprawy Mat.) 504, 1-88 (2014).

[2] Andreieva, D.M., Ignatovich, S.Yu.: Homogeneous approximation for minimal realizations of series of iterated integrals. Visnyk of V.N.Karazin Kharkiv National University, Ser. Mathematics, Applied Mathematics and Mechanics 96, 23-39 (2022).

On a class of finite-time stabilizing controls for nonlinear systems in a critical case

Maxim Bebiya, *Kharkiv, Ukraine*
Valerii Korobov, *Kharkiv, Ukraine*

We address the finite-time stabilization problem for nonlinear systems in a critical case. Namely, we study the following nonlinear system

$$\begin{cases} \dot{x}_1 = u, \quad |u(x)| \le d, \\ \dot{x}_i = c_{i-1}x_{i-1}^{2k_{i-1}+1} + f_{i-1}(t,x,u), \quad i = 2,\ldots,n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control, $d > 0$ is a given number, $k_i = \frac{p_i}{q_i}$ ($p_i > 0$ is an integer number, $q_i > 0$ is an odd number), $c_i \neq 0$ are real numbers, $f_i(t,x,u)$ are continuous functions, $f_i(t,0,0) = 0$ for all $t \ge 0$ ($i = \overline{1, n-1}$).

We solve the bounded control synthesis problem for system (1), which is to find a control $u = u(x)$ such that

(i) for every $x_0 \in U(0) \subset \mathbb{R}^n$ there exists a number $T(x_0) < +\infty$ such that $\lim\limits_{t \to T(x_0)} x(t,x_0) = 0$, where $x(t,x_0)$ is a solution of system (1) with $u = u(x)$ that satisfies the condition $x(0,x_0) = x_0$;

(ii) the control $u(x)$ satisfies the restriction $|u(x)| \le d$ for all $x \in \mathbb{R}^n \setminus \{0\}$.

We develop the results of [1], [2] to construct a class of bounded finite-time stabilizing controls $u = u_\alpha(x)$, $\alpha \ge 1$ to ensure finite-time convergence of the trajectories. Our approach is based on the controllability function method [3]. We introduce a family of controllability functions $\Theta_\alpha(x)$ to guaranty the inequality $\dot{\Theta}(x) \le -\beta\Theta^{1-\frac{1}{\alpha}}(x)$, $\alpha \ge 1$ for some $\beta > 0$ (which is sufficient for finite-time convergence). We formulate growth conditions on $f_i(t,x,u)$ under which we achieve finite-time convergence.

[1] Bebiya, M. O. and Korobov, V. I.: On Stabilization Problem for Nonlinear Systems with Power Principal Part. Journal of Mathematical Physics, Analysis, Geometry. 12, 113-133 (2016).

[2] Bebiya, M. O.: Global synthesis of bounded controls for systems with power nonlinearity. Visnyk of V.N. Karazin Kharkiv National University, Ser. Mathematics, Applied Mathematics and Mechanics. 81, 36-51 (2015).

[3] Korobov, V.I.: A general approach to the solution of the bounded control synthesis problem in a contollability problem. Math USSR Sb. 37, 535-557 (1980).

The scheme for the construction of solutions for the nonlinear boundary-value problem unsolved with respect to the derivative

Peter Benner, *Magdeburg, Germany*
Sergey Chuiko, *Magdeburg, Germany*
Olga Nesmelova, *Sloviansk, Ukraine*

We establish constructive necessary and sufficient conditions of solvability and a scheme for construction of the solutions of a nonlinear periodic boundary-value problem for a Rayleigh-type equation unsolved with respect to the derivative [1, 2].

The relevance of studying nonautonomous boundary-value problems, unsolved with respect to derivative, is also, associated with the fact that the study of traditional problems, resolved by derivative, sometimes complicated, for example, in the case of nonlinearities, not integrable in elementary functions. We consider the critical case where the equation for the generating amplitudes of a weakly nonlinear periodic boundary-value problem for a Rayleigh-type equation does not turn into an identity. For finding the constructive conditions for the solution and convergent iterative scheme for constructing approximate solutions to a nonautonomous nonlinear boundary-value problem unsolved with respect to the derivative we use the least squares method [3].

As an example of application of the proposed iterative scheme, we find approximations to the solutions of periodic boundary-value problems unsolved with respect to the derivative in the case of periodic problem for the equation used to describe the motion of satellites on elliptic orbits.

[1] Samoilenko, A.M., Chuiko, S.M.,. Nesmelova, O.V.: Nonlinear boundary-value problem unsolved with respect to the derivative, Ukrainian Math. Journ., 72(8), 1280–1293 (2021).

[2] Shlapak, Yu. D.: Periodic solutions of nonlinear second-order equations which are not solved for the highest derivative, Ukrainian Math. Journ., 26, № 6, 850–854 (1974).

[3] Benner, P., Chuiko, S., Nesmelova, O.: Least-squares method in the theory of nonlinear boundary-value problems unsolved with respect to the derivative, Ukrainian Math. Journ., 75(1), 40-55 (2023).

Adomian decomposition method for the nonlinear boundary-value problems

Peter Benner, *Magdeburg, Germany*
Sergey Chuiko, *Slavyansk, Ukraine*
Mykyta Popov, *Slavyansk, Ukraine*

We proves the existence of a solution of a nonlinear periodic boundary-value problem for an ordinary differential equation with switchings [1] and constructs an iterative scheme for finding the solution of this problem using the Adomian decomposition method [2, 3]. The relevance of the study of the boundary-value problem with switches is related to the wide application of similar problems in the study of non-isothermal chemical reactions [4]. We give an example of modelling such reactions. An example of finding approximations to the periodic solution of this problem will be given using the iterative scheme we have built.

The main problem of this study, is that when constructing solutions of nonlinear boundary-value problems, the problem of the impossibility of finding solutions in elementary functions arises, which, in turn, leads to large errors in the solutions of nonlinear boundary-value problems. A similar problem was demonstrated in our earlier papers for the periodic problem for the equation that defines the motion of a satellite in an elliptical orbit. Taking into account the above, the simplification of calculations of derivative nonlinearities and the possibility of finding solutions of nonlinear boundary-value problems, in particular, periodic boundary-value problems, in elementary functions can be achieved using the Adomian decomposition method. An example of such a simplification obtained.

[1] Benner, P., Chuiko, S., Zuyev, A.: A periodic boundary value problem with switchings under nonlinear perturbations. Boundary Value Problems, 50, 1-12 (2023).

[2] Adomian, G.: Convergent series solution of nonlinear equations. Journal of Computational and Applied Mathematics 11, 225–230 (1984).

[3] Chuiko S., Chuiko, A., Popov, N.: Adomian decomposition method in theory of nonlinear periodical boundary value problems. Nonlinear Oscillations 25 (4), 413–425 (2022).

[4] Benner, P., Seidel-Morgensternand, A., Zuyev, A.: Periodic switching strategies for an isoperimetric control problem with application to nonlinear chemical reactions. Applied Mathematical Modelling 69(1), 287–300 (2019).

Extended set of solutions of a bounded finite-time stabilization problem via V.I. Korobov's controllability function

Abdon E. Choque-Rivero, *Morelia, Mich., Mexico*

For the canonical system, an extended set of bounded finite-time stabilizing positional controls is proposed. V.I. Korobov's controllability function [3, 4, 5, 6] is used to construct these controls, which are dependent on a parameter.The mentioned extension is based on the enlarging of the interval of the mentioned parameter. We enlarge the parameter interval and explicitly compute its endpoints as functions of the dimension n of the given system. The constructed controllability function is exactly the motion time from the initial point to the origin. We consider the case when the controllability function is a non-unique solution of a certain equation. See [1, 2].

[1] Choque-Rivero, A.E.: Extended set of solutions of a bounded finite-time stabilization problem via the controllability function. IMA J. Math. Control Inform. 38(4), 1174–1188 (2021).

[2] Choque-Rivero, A.E.: Korobov's controllability function as motion time: Extension of the solution set of the synthesis problem. Accepted to appear in Mat. Fiz. Anal. Geom. 1–31 (2023).

[3] Choque Rivero A.E., Korobov V.I. and Skoryk V.O.: Controllability function as time of motion. I, Mat. Fiz. Anal. Geom. **11**(2) 208–225 (2004). English translation in http://arxiv.org/abs/1509.05127.

[4] Korobov, V.I.: A general approach to the solution of the problem of synthesizing bounded controls in a control problem, Mat. Sb. 109 (151) 582–606 (1979). English transl.: Mat. Sb. 37 (4) 535–557 (1980).

[5] Korobov, V.I.: Controllability function method, NITS, Inst. Comp. Research (2007).

[6] Korobov, V.I. and Sklyar G.M.: Methods for constructing of positional controls and an admissible maximum principle, Differ. Uravn. 26(11) 1914–1924 (1990).

Nonlinear autonomous boundary value problem for differential algebraic system

Sergey Chuiko, *Magdeburg, Germany*
Daria Diachenko, *Slavyansk, Ukraine*

We denote A and B are $(m \times n)-$measurable matrices and $Z(z, \varepsilon)$ is n measurable vector function. We establish constructive conditions of solvability and a scheme of construction of the solutions [1]

$$z(t, \varepsilon): \ z(\cdot, \varepsilon) \in \mathbb{C}^1[a, b(\varepsilon)], \ z(t, \cdot) \in \mathbb{C}[0, \varepsilon_0], \quad b(0) := b^*$$

for a nonlinear boundary-value problem for a nondegenerate differential algebraic system

$$A\,z' = B\,z + \varepsilon Z(z, \varepsilon), \quad \ell z(\cdot, \varepsilon) = \alpha. \tag{1}$$

Here, $\ell z(\cdot, \varepsilon)$ is a linear bounded vector functional

$$\ell z(\cdot, \varepsilon) : \mathbb{C}[a, b(\varepsilon)] \to \mathbb{R}^q.$$

We seek solutions of the problem (1) in a small neighborhood of the solution

$$z_0(t) \in \mathbb{C}^1[a, b^*]$$

of the generating Noether $(q \neq n)$ differential-algebraic boundary-value problem

$$A\,z_0' = B\,z_0, \quad \ell z_0(\cdot) = \alpha \in \mathbb{R}^q.$$

We assume the vector function $Z(z, \varepsilon)$ is a continuously differentiable with respect to the unknown $z(t, \varepsilon)$ in a small neighborhood of the solution of the generating problem and continuously differentiable with respect to the small parameter ε in a small positive neighborhood of zero. The matrix A is generally assumed to be rectangular $m \neq n$, or square, but degenerate [2]. We propose a convergent iterative algorithm for finding approximate solutions of the nonlinear autonomous boundary-value problem for a for a nondegenerate differential algebraic system (1).

[1] Chuiko, S.M., Boichuk, I.A.: An autonomous Noetherian boundary value problem in the critical case, Nonlinear Oscillations (N.Y.), 12, 405-416 (2009).

[2] Chuiko, S.M.: A generalized Green operator for a linear Noetherian differential-algebraic boundary value problem. Siberian Advances in Mathematics, 30, 3. 177-191 (2020).

Approximations of the solutions of nonlinear matrix equations using the Newton-Kantorovich method

Sergey Chuiko, *Slaviansk, Ukraine*
Kateryna S. Shevtsova, *Slaviansk, Ukraine*

The study of nonlinear matrix equations [1], in particular, the algebraic matrix Riccati equation [2, 3], is connected with numerous applications of such equations while solving the differential matrix Riccati equation, in the theory of nonlinear oscillations, in mechanics, biology, and radio technology, the theory of control and stability of motion, and others [4].

Newton's method [5] is applicable for finding approximations for solutions of nonlinear matrix equations in the case of unknown square matrix. To find approximations for the solutions of nonlinear matrix equations in the case of unknown rectangular matrix, the Newton-Kantorovich method [6, 7] used. As an example of the iterative scheme construction, approximations for the solutions of the nonlinear algebraic matrix Riccati equation and their accuracy errors were determined [8].

[1] Boichuk, A.A.: Criterion of the solvability of matrix equations of the Lyapunov type. Ukrainian Math. Journal, 50 (8), 1162-1169 (1998).

[2] Kuvshinov, V.M.: Specific features of the numerical solution of the matrix algebraic Riccati equation using the settling method. Uchenye Zapiski TsAGI, X(1), 69–87 (1979).

[3] Palin, V.V.: Solvability of Quadratic Matrix Equations. Vestnik Moscow University, Matematika. Mekhanika, 63 (6), 36–41 (2008).

[4] Boichuk, A.A.: A Critical Periodic Boundary Value Problem for a Matrix Riccati Equations. Differential Equations, 37 (4), 464–471 (2001).

[5] Kantorovich, L.V., Akilov, G.P.: Functional Analysis, Nauka, Moscow (1977).

[6] Chuiko, S.M.: To the generalization of the Newton-Kantorovich theorem. Visnyk of V.N. Karazin Kharkiv National University. Ser. Mathematics, Applied Mathematics and Mechanics, 85(1), 62–68 (2017).

[7] Chuiko, S.M.: To the generalization of the Newton-Kantorovich theorem in the Banakh space. Dopovidi NAS of Ukraine, 6 (2018).

[8] Chuiko, S.M., Shevtsova, S.K.: Solvability conditions for nonlinear matrix equations. Journal of Mathematical Sciences, 270, 3, 407-419 (2023).

Determining the speed of rotation of the rotor in case of incomplite information about the operation of two-phase asynchronous motor

Iryna Dmytryshyn, *Slovyansk, Ukraine*

Mathematical modeling of electromechanical systems as one of the stages of designing systems of an automated electrical device serves to study the behavior of the system in artificially designed emergency situations by blocking the measurement of certain components of the model. The estimation of the state of the parameters of the model, which ensures the stability of the process, has not yet been solved. Since the measurement of the flux linkage vector is not a simple procedure, it is necessary to use methods that allow determining the flux linkage of the rotor through dynamic equations by measuring the stator voltage, rotor speed and phase currents. The greatest interest in practice is the evaluation of the speed of rotation of the rotor and the torque.

The paper considers a mathematical model of a two-phase asynchronous motor [1].

$$\begin{aligned}
\dot{y}_1 &= -a_1 y_1 + a_2 U_1 + a_1 \mu x_1 + a_1 y_3 x_2,\\
\dot{y}_2 &= -a_0 y_2 + a_2 U_2 - a_1 y_3 x_1 + a_1 \mu x_2,\\
\dot{y}_3 &= a_3 y_2 x_1 - a_3 y_1 x_2 - x_3,\\
\dot{x}_1 &= a_4 y_1 - \mu x_1 - y_3 x_2,\\
\dot{x}_2 &= a_4 y_2 + y_3 x_1 - \mu x_2,\\
\dot{x}_3 &= 0,
\end{aligned} \tag{1}$$

The following notations are introduced in the studied model: $x = (\lambda_a, \lambda_b, \frac{n_p \cdot \tau_L}{I_m})^T, y = (i_a, i_b, n_p \cdot \omega)^T$, where i_a, i_b describe the stator currents, λ_a, λ_b – rotor fluxes, ω – speed of rotation of the rotor, U_1, U_2 – stator voltage, n_p – the number of pairs of poles, I_m – moment of inertia and τ_L – rotor torque, $a_0, a_1, a_2, a_3, a_4, \mu > 0$ – some constants.

In the work, a nonlinear observer for the unknown components x_3, y_3 is constructed using the method of invariant transformations.

[1] Sassano, M: Towards constructive nonlinear control systems analysis and design. PhD thesis, Control and Power Research Group Department of Electrical and Electronic Engineering Imperial College London (2001).

On controllability problems for the heat equation with variable coefficients on a half-axis controlled by the Neumann boundary condition

Larissa Fardigola, *Kharkiv, Ukraine*
Kateryna Khalina, *Kharkiv, Ukraine*

Consider the following control system:

$$w_t = \frac{1}{\rho}(kw_x)_x + \gamma w, \qquad x \in (0,+\infty),\ t \in (0,T), \tag{1}$$

$$\left(\sqrt{\frac{k}{\rho}} w_x\right)\Bigg|_{x=0} = u, \qquad t \in (0,T), \tag{2}$$

$$w(\cdot,0) = w^0, \qquad x \in (0,+\infty), \tag{3}$$

where $T > 0$ is a constant; ρ, k, γ, and w^0 are given functions; $u \in L^\infty(0,T)$ is a control. We assume $\rho, k \in C^1[0,+\infty)$ are positive on $[0,+\infty)$, $(\rho k) \in C^2[0,+\infty)$, $(\rho k)'(0) = 0$. We also assume ρ, k, γ satisfy some additional smoothness and growth conditions. The control system is considered in modified Sovolev spaces.

We prove that any initial state of the control system (except the zero one) is not null-controllable in a given time $T > 0$.

We also prove, however, that each initial state of the control system is approximately controllable to any target state in a given time $T > 0$.

Due to transformation operator generated by the equation data ρ, k, γ (see [1], [2]), the main results are obtained from their analogues obtained earlier in the case of constant coefficients for $\rho = k = 1$, $\gamma = 0$ (see [3]). Applying this operator is a focal point of the work. The results are illustrated by examples.

[1] L.V. Fardigola, *Transformation operators in controllability problems for the wave equations with variable coefficients on a half-axis controlled by the Dirichlet boundary condition,* Math. Control Relat. Fields **5** (2015), 31–53.

[2] L.V. Fardigola, *Transformation operators and modified Sobolev spaces in controllability problems on a half-axis,* J. Math. Phys., Anal., Geom. **12** (2016), 17–47.

[3] L. Fardigola and K. Khalina, *Controllability problems for the heat equation on a half-axis with a bounded control in the Neumann boundary condition,* Math. Control Relat. Fields **1** (2021), 211–236.

Problem of optimal exact observability of a Timoshenko beam

Mateusz Firkowski, *Szczecin, Poland*
Jarosław Woźniak, *Szczecin, Poland*

We consider the problem of optimal exact observability of distributed parameter system of clamped–free vibrating Timoshenko beam system governed by

$$\begin{cases} \ddot{w}(x,t) - w''(x,t) - \xi'(x,t) = 0, \\ \ddot{\xi}(x,t) - \xi''(x,t) + w'(x,t) + \xi(x,t) = 0, \end{cases}$$

for $x \in (0,1)$ and $t > 0$, with boundary conditions of the following form

$$\begin{cases} w(0,t) = \xi(0,t) &= 0, \\ w'(1,t) + \xi(1,t) = \xi'(1,t) &= 0. \end{cases}$$

We observe the deflection of the center line of the beam at the free end, i.e. $y = w(1,\cdot)$.

Then, we present some important facts about spectral properties of the operator of motion of the considered system. Next, we prove that it is not exactly observable in default topologies, and we find a stronger topology for state observation for which the system in question becomes exactly observable.

The main result of the talk is devoted to find optimal topology of the observable space. The sharpness of the obtained result is proved.

[1] Sklyar, G. M., Woźniak, J., Firkowski, M.: Exact observability conditions for Hilbert space dynamical systems connected with Riesz basis of divided differences. Syst. Control Lett. 145 (2020), 104782.

[2] Woźniak, J., Firkowski, M.: Sobolev's type optimal topology in the problem of exact observability for Hilbert space dynamical systems connected with Riesz basis of divided differences, (under review).

Partial differential equations in the module of copolynomials in several variables over a commutative ring

Sergey Gefter, *Kharkiv, Ukraine*
Aleksey Piven', *Kharkiv, Ukraine*

Let K be an arbitrary commutative integral domain with identity, $K[x_1, ..., x_n]$ be the ring of polynomials with coefficients in K and $K[x_1, ..., x_n]'$ be the module of K-linear mappings from $K[x_1, ..., x_n]$ to K. By a copolynomial over the ring K we mean an element of the module $K[x_1, ..., x_n]'$. For any multi-index $\alpha = (\alpha_1, ..., \alpha_n) \in \mathbb{N}_0^n$ the derivative $D^\alpha T = \frac{\partial^{|\alpha|}T}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\cdots\partial x_n^{\alpha_n}}$ $(|\alpha| = \sum\limits_{j=1}^{n} \alpha_j)$ of a copolynomial T is defined in the same way as in the classical theory of generalized functions: $(D^\alpha T, p) = (-1)^{|\alpha|}(T, D^\alpha p), \quad p \in K[x_1, ..., x_n]$. We prove an existence and uniqueness theorem for a differential equation of infinite order which can be considered as an algebraic version of the classical Malgrange-Ehrenpreis theorem for the existence of fundamental solutions for differential operators with constant coefficients.

Theorem 1 *Let $\mathcal{F} = \sum\limits_{|\alpha|=0}^{\infty} a_\alpha D^\alpha$ be a differential operator of infinite order on $K[x_1, ..., x_n]'$ with coefficients $a_\alpha \in K$. If a_0 is an invertible element of K, then for any copolynomial $T \in K[x_1, ..., x_n]'$ there exists a unique solution $u \in K[x_1, ..., x_n]'$ of the differential equation $\mathcal{F}u = T$.*

The *Cauchy-Stieltjes transform* of a copolynomial $T \in K[x_1, ..., x_n]'$ is defined as the following formal Laurent series from the ring $\frac{1}{s_1 s_2 \cdots s_n} K[[\frac{1}{s_1}, \frac{1}{s_2}, ..., \frac{1}{s_n}]]$:

$$C(T)(s) = \sum_{|\alpha|=0}^{\infty} \frac{(T, x^\alpha)}{s^{\alpha+\iota}}, \quad s = (s_1, ..., s_n),\ x = (x_1, ..., x_n),\ \iota = (1, ..., 1).$$

The mapping $C : K[x_1, ..., x_n]' \to \frac{1}{s_1 s_2 \cdots s_n} K[[\frac{1}{s_1}, \frac{1}{s_2}, ..., \frac{1}{s_n}]]$ is an isomorphism of K-modules. The multiplication of copolynomials is defined through the multiplication of their Cauchy-Stieltjes transforms.

Let $P \in K[z_1, ..., z_m]$, $P(0) = 0$ and let $\mathcal{F}_j = \sum\limits_{|\alpha|=0}^{\infty} a_{j,\alpha} D^\alpha$ $(j = 1, ..., m)$ be differential operators in $K[x_1, ..., x_n]'$ with coefficients $a_{j,\alpha} \in K$. Consider the following Cauchy problem in the module $K[x_1, ..., x_n]'[[t]]$ of formal power series of the form $u(t,x) = \sum\limits_{k=0}^{\infty} u_k(x)t^k$ with coefficients $u_k(x) \in K[x_1,...,x_n]'$:

$$\frac{\partial u(t,x)}{\partial t} = P\left((\mathcal{F}_1 u)(t,x),...,(\mathcal{F}_m u)(t,x)\right),\ u(0,x) = Q(x) \in K[x_1,...,x_n]'. \tag{1}$$

Theorem 2 *Let K contains the field of rational numbers. Then for any copolynomial $Q \in K[x_1,...,x_n]'$ the Cauchy problem (1) has a unique solution.*

This work was supported by the Akhiezer Foundation.

Continual distribution for the Bryan–Pidduck equation

Vyacheslav Gordevskyy, *Kharkiv, Ukraine*
Oleksii Hukalov, *Kharkiv, Ukraine*

The nonlinear integrodifferential Boltzmann equation [1] that describes the evolution of rarefied gases is one of the main equations of the kinetic theory of gases. For the rough sphere model, it is called the Bryan-Pidduck equation. We obtain a so-called continual distribution for the global Maxwellian (it depends only on the linear and angular velocities of gas particles) in the form:

$$f(t,x,V,\omega,u) = \int_{\mathbb{R}^3} \varphi(t,x,u) M(V,\omega,u) du, \tag{1}$$

where the Maxwellian $M(V,\omega,u)$ is given by the formula [1]:

$$M(V,\omega,u) = \rho I^{3/2} \left(\frac{\beta}{\pi}\right)^3 e^{-\beta\left((V-u)^2 + I\omega^2\right)}. \tag{2}$$

As a measure of deviation between the parts of Bryan-Pidduck equation, we use a uniform integral error. In the case of the model of rough spheres, it has the form:

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f,f) \Big|. \tag{3}$$

In the article [2], we constructed the sought approximate solution (1). We established sufficient conditions for the coefficient function $\varphi(t,x,u)$ and hydrodynamic parameters appearing in the distribution (2), which enable one to make the analyzed error (3) as small as desired.

[1] S. Chapman, T.G. Cowling: The mathematical theory of non-uniform gases, Cambridge Univ. Press, Cambridge (1952).

[2] V.D. Gordevskyy, O.O. Hukalov: Continual Distribution for the Bryan – Pidduck Equation. Ukrains'kyi Matematychnyi Zhurnal 11, 1487-94 (2020).

Mathematical simulation of cold extrusion processes with complex tool configuration

Natalia Hrudkina, *Zaporizhzhya, Ukraine*

Cold extrusion processes provide a high surface quality and precise dimensions of stamped workpieces and parts and demonstrate a steady trend to expansion of technological capabilities and implementation in manufacturing [1]. The configuration of the tool (the presence of roundings) allows to form the required profile of the part and significantly affects on the deformation and power modes of the deformation [2], [3]. Determination of the optimal power mode in the form of engineering formulas, taking into account the influence of design features of the tool, will contribute to a more active implementation of these processes in the manufacturing. Proposes the using of an approximate curve as a replacement for a quarter of a circle reflecting of the matrix rounding. Developed new kinematic module with rounding allows to expand the capabilities of upper bound method for modeling the processes of cold extrusion with a complex tool shape [3]. This will allow in the future to use the above calculations in new schemes and will help to obtain an assessment of the power mode and shape resizing and, as a result, to develop recommendations for the optimal configuration of the tool and more active implementation of these processes in the manufacturing.

[1] Bhaduri A.: Extrusion. Mechanical Properties and Working of Metals and Alloys., 599-646 (2018).

[2] Hrudkina N. S., Aliieva L. I.: Modeling of cold extrusion processes using kinematic trapezoidal modules. FME Transactions. 48(2), 357-363 (2020).

[3] Hrudkina N., Aliieva L., Markov O., Kartamyshev D., Shevtsov S., Kuznetsov M.: Modeling the process of radial-direct extrusion with expansion using a triangular kinematic module. Eastern European Journal of Enterprise Technologies. 3/1 (105), 17–22 (2020).

Levels in the gap of the essential spectrum of a differential operator

Aleksandr Kholkin, *Dnipro, Ukraine*

Various approaches to the study of discrete levels in spectral gaps were considered in works of Rofe-Beketov F., Kholkin A., Gestesi F., Simon B. and Teshl G.

Consider a self-adjoint differential equation of order $r \geq 1$ with operator coefficients

$$l\,[y] = \sum_{k=0}^{r} i^k\, l_k\,[y] = \lambda\, W\,(x)\, y, \tag{1}$$

$$l_{2j} = D^j p_j\,(x)\, D^j,\ \ {p_j}^*\,(x) = p_j\,(x)\,,\ \ l_{2j} = D^j p_j\,(x)\, D^j,\ \ {p_j}^*\,(x) = p_j\,(x)\,,$$

operator coefficients $p_j\,(x)\,,\ q_j\,(x)$ uniformly continuously depend on x together with its derivatives up to the order $2r$ inclusive, operator $W\,(x)$ and the coefficient of the highest derivative in equation (1) has a bounded inverses in separable Hilbert space H for $x \in (a, b)$. By L denote minimal differential operator generated by a differential expression $l_W\,[y] = W^{-1}\,(x)\; l\,[y]$. Operation $l_W^2[y] = l_W\,[l_W[y]]$ can be considered as an ordinary differential order $2r$. By M denote the minimal differential operator generated by the operation $l_W^2[y]$. Let $\tilde{L}$ self-adjoint operator extension of L. Operator $\left(\tilde{L}\right)^2$ is a self-adjoint extension $\tilde{M}$ of the positive symmetric operator M. By M_b we denote the restriction of $\tilde{M}$ by minimality requirement regarding b, $p = Def\left\{\tilde{M}\,\middle|\, D\left(\tilde{M}\right) \cap D\left(M_b^F\right)\right\}$, $N\,(\lambda, \mu)$ – number of eigenvalues $\lambda_k \in (\lambda, \mu)$ operator $\tilde{L}$ counting multiplicities $æ\,(\lambda_k)$.

Let $Y(x, \lambda)$ be a fundamental solution of the problem (1), $U_a\,[y] = 0$. Let

$$Y(x, \lambda, \mu) = \{Y(x, \lambda); Y(x, \mu)\}\,,$$

$$Y^{\Delta}(x, \lambda, \mu) = \mathrm{col}\left\{Y(x, \lambda, \mu); Y'(x, \lambda, \mu); \ldots; Y^{(r-1)}(x, \lambda, \mu)\right\}.$$

Theorem 1 *Let (α, β)– gap in the essential spectrum of the operator $\tilde{L}$, $\alpha < \lambda < \mu < \beta$. Then*

$$N(\lambda, \mu) - p \leq \sum_{x \in (a,b)} \mathrm{nul} Y^{\Delta}(x, \lambda, \mu) \leq N(\lambda, \mu) \tag{2}$$

If $æ_{L_b}\,(\lambda) = æ_{L_b}\,(\mu) = 0$, where $L_b \subseteq \tilde{L}$ is the minimal operator with respect to the end b, then in (2) instead of p we can take $\min\,\{p,\, Def\, M_b - æ\,(\lambda) - æ\,(\mu)\}$. *If $a > -\infty$, then the theorem is also true for $\lambda = \alpha$, $\mu = \beta$.*

Mathematical maintenance to design the automated control systems in agreement with the circular economy principles

Irina Kolupaieva, *Kharkiv, Ukraine*
Igor Nevliudov, *Kharkiv, Ukraine*
Yurii Romashov, *Kharkiv, Ukraine*

The mathematical maintenance is considered as the set of the mathematical objects and the rules for their applications to design the automated control systems. Such consideration allows us to imagine designment results of the automated control systems as predefined by the used mathematical maintenance, so it is led to development of such mathematical maintenance as the actually important problem connected with improvement of the automation control systems. Implementation of the circular economy is considered at resent as the way of resolving a lot of global modern challenges especially about environment pollution, but to realise this way it is necessary to rebuild a lot of conventional approaches in different fields, including in designment of the automated control systems.

It is proposed to make more wider the conventional approaches to design the automated control systems on the basis of improved mathematical modelling of the processes in the automated systems to consider the different kinds of wastes during exploitation. Application of the the parametric identification procedure will allow to represent the mathematical model of the complicated automation objects by means the ordinary differential equations suitable for designment of the automated control systems. Basing on the existed experience [1, 2], it is assumed, that such detail mathematical modelling of the automation objects will allow us to design the automated control systems providing the exploitation with minimum wastes in agreement with the circular economy principles.

[1] Mamalis, A.G., Nevliudov, I., Romashov, Yu.: An approach for numerical simulating and processing of measured electrical signals from board sensors installed on wheeled electro-mechanical platforms. Journal of Instrumentation. 16(10), P10006 (2021).

[2] Alyokhina, S., Nevliudov, I., Romashov, Yu.: Safe Transportation of Nuclear Fuel Assemblies by Means of Wheeled Robotic Platforms. Nuclear and Radiation Safety. 3(91), 43-50 (2021).

On solving the controllability problem in the case of a positive constrained control

Valerii Korobov, *Kharkiv, Ukraine*

Let us consider the linear controllable system

$$\dot{x} = Ax + Bu, \quad u \in \Omega. \tag{1}$$

The synthesis problem is solved, which is to construct a control $u(x) \in \Omega$ that transfers an arbitrary point x from the neighborhood V of the origin to the origin in a finite time. The control $u(x)$ is found using the controllability function method. Moreover, the controllability function is found as the time of motion from the point $x \in V$ to the origin.

Starting from the works [1, 5], various controllability criteria were considered for different kinds of constraints on the control. The case of constraints with the condition $0 \in \Omega$ is considered in [4]. In the case of general constraints on the control, that is without the requirement $0 \in \Omega$, necessary and sufficient conditions are given in [3]. In addition to conditions from [4], the return condition to the origin on some interval $[t_1, t_2]$ is required. But controllability criteria do not provide an explicit formula for control.

In this talk we assume that the control domain Ω does not contain 0 as an interior point; moreover, the point 0 may not belong to the set Ω. We give an explicit formula for the control using the controllability function method.

Also, the case when the linear controllable system has a non-autonomous term is considered.

[1] Brammer R., Controllability in linear autonomous systems with positive controllers, SIAM J. Control 10, 339–353 (1972)

[2] Korobov V. I. The method of controllability function, R&C Dynamics, 2007: 1-576.

[3] Korobov V.I. Geometric criterion for controllability under arbitrary constraints on the control. J Optim Theory Appl 134, 161-176 (2007)

[4] Korobov V. I., Marynych A. P., Podolskii E. N. Controllability of linear autonomous systems in the presence of constraints on the control, Differ. Equ., 15 1136–1142 (1980)

[5] Saperstone S.H., Yorke J.A. Controllability of linear oscillatory systems using positive controllers, SIAM Journal on Control, 10, 2(1971), 253-262

On perturbations range in the feedback synthesis problem for robust linear system

Valerii Korobov, *Kharkiv, Ukraine*
Tetiana Revina, *Kharkiv, Ukraine*

We consider the synthesis problem for an autonomous linear system with continuous *bounded unknown perturbations*:

$$\dot{x} = (A + R(t, x))x + Bu, \tag{1}$$

where $t \geq 0,\ \ x \in Q \subset \mathbb{R}^n$, Q is a neighborhood of the origin; $u \in \mathbb{R}^r$ is a control satisfying the constraint $\|u\| \leq d$; A and B are given constant matrices, $R(t, x) = (r_{ij}(t, x))_{i,j=1}^n$.

We assume that the system $\dot{x} = Ax + Bu$ is completely controllable.

We assume that functions $r_{ij}(t, x)$ are *unknown functions* and $\max\limits_{i,j} \mid r_{ij}(t, x) \mid \leq \Delta$ for all $(t, x) \in [0, +\infty) \times Q$.

The problem is to construct a *bounded control* which does not depend on perturbation and steers an initial point $x_0 \in Q$ to the origin in a *finite time (settling-time function)* for any perturbations satisfying the constraint. Also the problem is to find Δ.

Our approach is based on the Controllability Function Method proposed by V. I. Korobov [1, 2]. The results show that the control designed by the Controllability Function Method can guarantee that the trajectory of the system steers to the origin in a finite time under perturbations. This study shows the relations between the value of perturbations (i.e., Δ) and the upper bound of the settling-time function.

As an application of the results given in the talk, the feedback synthesis problem for a robust system in which the Controllability Function is specified explicitly is solved.

The work was carried out under the support of the N. I. Akhiezer Foundation.

[1] Korobov V. I. The method of controllability function, R&C Dynamics, 576 p (2007).

[2] Korobov V. I. Sklyar G. M. Methods for constructing positional controls, and a feasible maximum principle, Dif. Equ. 26(11) (1990), 1422–1431.

Numerical experiments with homogeneous approximation of nonlinear systems

Marcin Korzeń, *Szczecin, Poland*
Grigoriy Sklyar, *Szczecin, Poland*
Jarosław Woźniak, *Szczecin, Poland*

The homogeneous approximation is a certain kind of simplification of a nonlinear control system that makes it easier to integrate and also easier to solve different controllability tasks. A homogeneous approximation simplifies a given system and maintains its main properties. Both systems are equivalent up to a nonlinear transformation. Having the approximation and transformation, we can compare the trajectories of both systems with the same control signals. During the talk, we would like to present the procedure of homogeneous approximation from a computational point of view, and we give the numerical experiments with some nonlinear control systems and their homogeneous approximations. Comparing the system trajectories, we would like to show and briefly discuss the quality of such approximation.

Existence of well-posedness value problem for the Helmholtz equation

Alexander Makarov, *Kharkiv, Ukraine*
Anna Chernikova, *Kharkiv, Ukraine*

The Helmholtz equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} + \Delta u(x,t) = ku(x,t), \quad x \in \mathbb{R}^n,$$

where $k \in \mathbb{R}$, has a significant impact in mathematical physics and can occur in electrodynamics and thermodynamics. But this equation is not Petrovsky well-posed. That is why the Cauchy problem is not well-posed in the Schwartz space $\mathcal{S}$ and in spaces of exponential growth functions.

Let us consider the boundary-value problem for this equation with the boundary conditions:

$$u(x,0) + bu(x,T) = \varphi_1(x), \quad u_t'(x,0) + bu_t'(x,T) = \varphi_2(x),$$

where $b > 0$. It is well-posed in the space $\mathcal{S}$ and in spaces of exponential growth functions as well. Moreover, if $k \geq 0$, the boundary value problem is parabolic. Therefore, its solutions are infinitely differentiable for $\varphi_j \in L_2(\mathbb{R}^n)$. Applying the Fourier transform with respect to spatial variables, we get the boundary value problem:

$$\begin{cases} \dfrac{\partial^2 \widetilde{u}(s,t)}{\partial t^2} - |s|^2 \Delta \widetilde{u}(s,t) = k\widetilde{u}(s,t), \\ \widetilde{u}(s,0) + b\widetilde{u}(s,T) = \widetilde{\varphi_1}(s), \quad \widetilde{u}_t'(s,0) + b\widetilde{u}_t'(s,T) = \widetilde{\varphi_2}(s). \end{cases}$$

The eigenvalues of the characteristic equation are $\lambda_{1,2}(s) = \pm\sqrt{k + |s|^2}$. If $k \geq 0$, the solution of the problem has the following form:

$$\widetilde{u}(s,t) = \frac{1}{(1 + be^{\lambda T})(1 + be^{-\lambda T})} \times \left((\operatorname{ch} \lambda t + b \operatorname{ch} \lambda (T-t)) \widetilde{\varphi_1}(s) + \frac{\operatorname{sh} \lambda t + \operatorname{sh} \lambda (T-t)}{\lambda} \widetilde{\varphi_2}(s) \right),$$

where $\lambda(s) = \sqrt{k + |s|^2}$. Since $\frac{\operatorname{ch} \lambda t}{\exp \lambda t} \leq 1$, the given solution belongs to the space $\mathcal{S}$ if $\varphi_j \in \mathcal{S}$. Moreover, since

$$|\widetilde{u}(s,t)| \leq B \left(\exp\left((t-T)\sqrt{|s|^2 + k} \right) + \exp\left(-t\sqrt{|s|^2 + k} \right) \right) \max_j |\varphi_j(s)|,$$

the solution u is infinitely differentiable if $\varphi_j \in L_2(\mathbb{R}^n)$.

The effective nilpotency orders in the special multi-flags

Piotr Mormul, *Warsaw, Poland*

Special multi-flags (recalled during previous DECT conferences) are locally nilpotentizable in Sussmann's sense. That is, weakly nilpotent in the more modern terminology. More precisely, the germs of rank-$(m+1)$ distributions generating special m-flags of given length r are stratified into so-called *singularity classes.* They are encoded by words of length r over the alphabet $\{1, 2, \dots, m, m+1\}$ starting with letter 1 and such that every letter $L > 1$ in a word has to its left a letter $L - 1$. (For instance, for $m \geq 3$ the word 1.2.1.4 is not allowed.) The detailed construction of the singularity classes has been given in [1] (for $m = 2$) and [2] (for general m). The number of singularity classes with fixed width m and length r is approximately $\frac{1}{(m+1)!}(m+1)^r$.

To each singularity class $\mathcal{C}$ there is associated its Lie algebra $l(\mathcal{C})$ generated over the reals by a local nilpotent vector fields' basis of the underlying distribution. The algebras $L(\mathcal{C})$ are nilpotent of precisely known *nilpotency orders.* The aim of the presentation is to give new informations about the old (going back to the year 2004) algorithm of computing those nilpotency orders.

A farther future objective would be to prove that all those algebras are pairwise non-isomorphic.

[1] Mormul P.: Singularity classes of special 2-flags. Symmetry, Integrability and Geometry: Methods and Applications 5, 102, 22 pages (2009).

[2] Mormul, P.: Singularity classes of special multi-flags, I. Proceedings of the International Geometry Center 16, 142-160 (2023).

Robust stability of a class of switched positive linear functional differential equations

Nguyen Khoa Son, *Hanoi, VietNam*
Le Van Ngoc, *Hanoi, VietNam*

This paper investigates the robustness of exponential stability of a class of positive switched systems, under arbitrary switching, described by linear functional differential equations (FDE) of the form

$$\dot{x}(t) = A^0_{\sigma(t)}x(t) + \int_{-h}^{0} d[\eta_{\sigma(t)}(\theta)]x(t+\theta),\ t \geq 0,\ \sigma \in \Sigma, \tag{1}$$

where Σ is a set switching σ which are piece-wise constant functions $\sigma : [0, +\infty) \to \{1, 2, \dots, N\}$. We will measure the stability robustness of such a system (which is considered as a nominal system) subject to parameter affine perturbations of its constituent subsystems matrices $A^0_k, \eta_k(\cdot), k = 1, 2, \dots, N$, by introducing the notion of structured stability radius. Some formulas for computing this radius, as well as estimating its lower bounds and upper bounds are established. In the case of switched linear systems with multiple discrete time-delays or/and distributed time-delays the obtained results yield tractably computable formulas or bounds for the stability radius. The extension of the obtained results to non-positive systems and the class of multi-perturbations has been presented. Examples are given to illustrate the proposed method.

[1] Liberzon, D.: Switching in Systems and Control. Berlin, Germany: Birkhäuser (2003).

[2] Son, N.K. and Ngoc, L.V.: Exponential stability analysis for a class of switched nonlinear time-varying functional differential systems. Nonlinear Analysis: Hybrid Systems. 44:101177 (2022).

Virus dynamics model with distributed delay

Alexander Rezounenko, *Kharkiv, Ukraine*

Qualitative properties of mathematical models of viral infections are studied. Such models attract much attention during last years, especially after wide spread of viral diseases, including COVID-19, HIV, hepatitis B and C. Many viruses continue to be a major global public health issues.

We are interested in a class of virus dynamics model with reaction-diffusion, logistic growth terms and a general non-linear infection rate functional response. The cases with and without immune response are discussed. The model consists of PDEs with delay(s), including the case of state-selective delay [1, 2]. The type of delay is a distributed analog to a discrete state-dependent delay. We first investigate conditions of well-posedness of the delay initial-value problem. Our main mathematical tool in studying of the asymptotic behaviour of solutions is the quasi-stability method developed by I.D. Chueshov [3]. We construct a dynamical system in a Hilbert space and prove the existence of a finite-dimensional global attractor. To prove the natural for a virus dynamics model dissipativness of the dynamical system we conduct a parallel study in a Banach space. The mentioned approach with the parallel study in different spaces was presented for a viral dynamics model in recent article [4].

[1] A. Rezounenko, J. Wu, A non-local PDE model for population dynamics with state-selective delay: Local theory and global attractors. Journal of Computational and Applied Mathematics, 190(1-2), 99-113, (2006). DOI: 10.1016/J.CAM.2005.01.047

[2] A. V. Rezounenko, Partial differential equations with discrete and distributed state-dependent delays. Journal of Mathematical Analysis and Applications, 326, 1031-1045 (2007). DOI: 10.1016/j.jmaa.2006.03.049

[3] I.D. Chueshov, Dynamics of Quasi-Stable Dissipative Systems, Springer, Cham, (2015), DOI: 10.1007/978-3-319-22903-4

[4] A. V. Rezounenko, Viral Infection Model with Diffusion and Distributed Delay: Finite-Dimensional Global Attractor. Qualitative Theory of Dynamical Systems, 22(1), 11 (2023). DOI: 10.1007/s12346-022-00707-6

On the Riesz basis property of the infinite-dimensional delay differential equations of the neutral type

Grigorij Sklyar, *Szczecin, Poland*
Piotr Polak, *Szczecin, Poland*
Bartosz Wasilewski, *Szczecin, Poland*

We present results extending some existing ones concerning the differential delay equations of the neutral type. Namely, we analyze the neutral delay system of the form

$$\dot{z}(t) = A\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\mathrm{d}\theta, \qquad (1)$$

where $z(t)$ takes values in a separable Hilbert space H. For the finite-dimensional case of $H = \mathbb{C}^n$ the system has been thoroughly studied in terms of stability and stabilizability in [1], [2] and other works. The main tool for stability and stabilizability analysis used in this works is the existence of a Riesz basis of subspaces consisting of $\mathcal{A}$-invariant subspaces where the operator $\mathcal{A}$, which represent the system (1), is of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} y(t) \\ z_t(\cdot) \end{pmatrix} = \mathcal{A}\begin{pmatrix} y(t) \\ z_t(\cdot) \end{pmatrix}, \ \mathcal{A}\begin{pmatrix} y \\ z(\cdot) \end{pmatrix} = \begin{pmatrix} \int_{-1}^{0} A_2(\theta)\dot{z}(\theta)\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(\theta)\mathrm{d}\theta \\ \mathrm{d}z(\theta)/\mathrm{d}\theta \end{pmatrix}$$

where $z_t(\cdot) = z(t+\cdot)$ and the domain of the operator $\mathcal{A}$ is given by

$$D(\mathcal{A}) = \{(y, z(\cdot)) : z(\cdot) \in H^1([-1,0]; H), y = z(0) - Az(-1)\} \subset$$
$$\subset H \times L^2([-1,0]; H).$$

The space $H \times L^2([-1,0]; H)$ is a Hilbert space. We will present some results concerning the existence of a Riesz basis of subspaces of the space $H \times L^2([-1,0]; H)$ consisting of $\mathcal{A}$-invariant subspaces for the more general separable Hilbert space H, thus extending existing results from [1].

[1] R. Rabah, G.M. Sklyar, A.V. Rezounenko, *Stability analysis of neutral type systems in Hilbert space*, J. Differ. Equ., **214** (2005), 391–428.

[2] R. Rabah, G.M. Sklyar, P.Y. Barkhayev, *Stability and stabilizability of mixed retarded-neutral type systems*, ESAIM: COCV **18** (3) (2012), 656–692.

Problem of linearizability for non-autonomous control systems

Jekatierina Sklyar, *Szczecin, Poland*
Grigorij Sklyar, *Szczecin, Poland*
Svetlana Ignatovich, *Kharkiv, Ukraine*

In the presentation we recall the classical concept of linearization problem for nonlinear control systems. Further we discuss the development of this concept to the case of non-autonomous systems obtained by the authors during the last years [1]–[4]. Namely, we consider the class of non-autonomous control systems of the form

$$\dot{x} = f(t,x) + g(t,x)u, \tag{1}$$

where $f(t,x)$ $g(t,x)$ are of the class $C^1([\alpha,\beta]\times Q)$, $Q\subset\mathbb{R}^n$, and $u\in\mathbb{R}^1$. We discuss conditions under which there exists a change of variables $y = F(t,x)\in C^2$ that reduces the system (1) to the linear form

$$\dot{y} = A(t)y + b(t)u,$$

where matrices $A(t)$, $b(t)$ are analytic on $[\alpha,\beta]$. One of the main steps of this study is to reduce system (1) to a driftless form

$$\dot{z} = \widetilde{g}(t,z)u,$$

which can be regarded as a canonical form suitable for both linear and nonlinear control systems, the same for both autonomous and non-autonomous cases.

[1] Sklyar, K.: On mappability of control systems to linear systems with analytic matrices. Systems Control Lett. 134 (2019), 104572.

[2] Sklyar, K., Ignatovich, S.: On linearizability conditions for non-autonomous control systems. Advances in Intelligent Systems and Computing, 2020, 1196 AISC, 625-637.

[3] Sklyar, K.V., Ignatovich, S.Yu.: Invariants of linear control systems with analytic matrices and the linearizability problem. Journal of Dynamical and Control Systems, 29 (2023), 111-128.

[4] Sklyar, K.V., Ignatovich, S.Y., Sklyar, G.M.: Linearizability problem and invariants for multi-input non-autonomous control systems. 2023 31st Mediterranean Conference on Control and Automation, MED 2023, 998-1003.

Modelling of the wave dynamics in a system for rope jumping and free falling

Mykyta Sobur, *Kharkiv, Ukraine*
Sergii Poslavskyi, *Kharkiv, Ukraine*

We consider discrete mathematical model of the rope system for jumping and free falling. A viscoelastic description of a climbing rope is used. The mechanical system under consideration includes two climbing ropes. One of them is so called base rope (static rope), which is mounted almost horizontally, and other is a leash (dynamic rope), to which the load is attached. The main interest for us is the wave dynamics of this system. Two types of waves can be observed: longitudinal and transverse. Their traveling and interaction influence the forces arising in the system. The design of the rope system is created usually in such a way as to minimize the maximum loads on the jumping person and on the fastening elements. The 2D model makes it possible to capture the characteristic features of the complex wave pattern observed during an experimental study of the braking of the movement of a falling load.

[1] Leuthausser U.: The physics of a climbing rope under a heavy dynamic load. Journal of sports engineering and technology. 231(2), 125-135. (2017).

[2] Christensen, R.M.: Theory of Viscoelasticity An Introduction. Academic Press, New York and London, (1971).

Localization of a solution to a mixed problem of PDE

Kateryna Stiepanova, *Kharkiv, Ukraine*
Daryna Shevchuk, *Kharkiv, Ukraine*

Let's consider the model representative of the problem:

$$u_t = (u^m)_{xx} \quad \forall (t,x) \in (0,T) \times (0,\infty), \quad m > 1, \quad T < \infty;$$

$$u(0,x) = u_0(x) \quad \forall x \in [0,\infty),$$
$$u(t,0) = f(t) \quad \forall t \in [0,T),$$
$$f(t) \to \infty \quad \text{at} \quad t \to T.$$

The last condition defines the limit regime with a sharpening (here, the sharpening time T). All known results on the boundary conditions with sharpening were obtained using a method based on the creation of barrier functions. These techniques limit the solution of the problem within a certain region and they are mostly associated with various explicit automodel solutions. However, this approach cannot be applied to equations that do not admit the corresponding equation theorems.

In this work, we consider a much more difficult Cauchy-Dirichlet problem, for which an exact sufficient condition for the localization of the solution. The proof of the effect of localization of the problem for a wide class of parabolic equations is based on special integral a priori or estimates that combine the ideas of [1]-[5].

[1] Antontsev, S. N. On the localization of solutions of nonlinear degenerate elliptic and parabolic equations, Sov. Math., Dokl. 1981. V. 24. P. 420-424.

[2] Diaz J. I., Veron L. Local vanishing properties of solutions of elliptic and parabolic quasilinear equations, Trans. Amer. Math. Soc. 1985. V. 290. No 2. P. 787-814.

[3] Oleinik O. A., Iosif'yan G. A. An analogue of Saint-Venant's principle and the uniqueness of solutions of boundary value problems for parabolic equations in unbounded domains, Russian Math. Surveys. 1976. V. 31. No 6. P. 153–178.

[4] Akulov V. F., Shishkov A. E. On asymptotic properties of solutions of mixed problems for quasilinear parabolic equations in unbounded domains, Math. USSR-Sb. 1992. V. 72. No 2. P. 557–567.

[5] Stiepanova K. V., Shishkov A. E. Strong and weakened localization of solutions to quasilinear parabolic equations, Reports of the National Academy of Sciences of Ukraine. 2013. V. 7. P. 30-36.

Return condition for oscillating systems with constrained positive control

Oleh Vozniak, *Kharkiv, Ukraine*
Valery Korobov, *Kharkiv, Ukraine*

In this paper we consider the constrained null-controllability problem with the assumption that the origin is not an equilibrium point of the system. For this problem different results were obtained [1]-[4]. In the paper [2] the concept of return condition on an interval and the corresponding criterion were introduced by V.I. Korobov. That condition means that for some interval I for any $T \in I$ we can construct a control $u_T(t)$ such that the trajectory starting from the origin can return there in the time T.

However checking this criterion and finding $u_T(t)$ can be rather difficult. We have considered the oscillating system

$$\dot{x}_{2k-1} = kx_{2k},\ \dot{x}_{2k} = -kx_{2k-1} + u, \quad k = 1, 2, ..., n, \tag{1}$$

with piecewise control $u_T(t)$ and constraints $u \in [c, 1]$ or $u \in \{c, 1\}, c > 0$. This problem can be written as the trigonometric momentum problem

$$\int_0^T u(t)e^{kit}\,dt = 0,\ k = 1, 2, ..., n. \tag{2}$$

In this paper several solutions were proposed. In particular that the control with only two switch points is enough to solve this problem for system of any size n:

$$u(t) = \begin{cases} \frac{1}{2}, & 0 \le t \le \alpha, \\ 1, & \alpha \le t \le 2\pi, \\ \frac{1}{2}, & 2\pi \le 2\pi + \alpha, \end{cases} \quad \text{or} \quad u(t) = \begin{cases} c, & 0 \le t \le \alpha, \\ 1, & \alpha \le t \le 2\pi, \\ 1 - c, & 2\pi \le 2\pi + \alpha. \end{cases} \tag{3}$$

[1] Bianchini, R. M.: Local Controllability, Rest States, and Cyclic Points, SIAM Journal on Control and Optimization, Vol. 21, pp. 714–720, 1983.

[2] Korobov, V.I.: Geometric Criterion for Controllability under Arbitrary Constraints on the Control. J Optim Theory Appl 134, 161–176 (2007). DOI: 10.1007/s10957-007-9212-2

[3] Margheri, A.: On the 0-local controllability of a linear control system. J Optim Theory Appl 66, 61–69 (1990). DOI: 10.1007/BF00940533

[4] Zverkin A. M., Rozova V. N.: Reciprocal controls and their applications, Differ. Uravn., 23:2 (1987), 228–236.

DECT 2023

Organizing Committee

Valerii Korobov (*V. N. Karazin Kharkiv National University, Ukraine*)

Grigorij Sklyar (*West Pomeranian University of Technology in Szczecin, Poland*)

Grigory Zholtkevych (*V. N. Karazin Kharkiv National University, Ukraine*)

Marcin Korzeń (*West Pomeranian University of Technology in Szczecin, Poland*)

Jekatierina Sklyar (*West Pomeranian University of Technology in Szczecin, Poland*)

Jaroslaw Wozniak (*West Pomeranian University of Technology in Szczecin, Poland*)

Piotr Polak (*West Pomeranian University of Technology in Szczecin, Poland*)

Svetlana Ignatovich (*V. N. Karazin Kharkiv National University, Ukraine*)

Participants

Daria Andreieva (*V. N. Karazin Kharkiv National University, Ukraine*)

Maxim Bebiya (*V. N. Karazin Kharkiv National University, Ukraine*)

Peter Benner (*Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*)

Anna Chernikova (*V. N. Karazin Kharkiv National University, Ukraine*)

Abdon E. Choque-Rivero (*Universidad Michoacana de San Nicolas de Hidalgo, Mexico*)

Sergiy Chuiko (*Donbass State Pedagogical University, Sloviansk, Ukraine; Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*)

Daria Diachenko (*Donbass State Pedagogical University, Sloviansk, Ukraine*)

Iryna Dmytryshyn (*Institute of Applied Mathematics and Mechanics, National Academy of Sciences of Ukraine, Sloviansk, Ukraine*)

Larissa Fardigola (*B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, Kharkiv, Ukraine;*

V. N. Karazin Kharkiv National University, Ukraine)

Mateusz Firkowski (*West Pomeranian University of Technology in Szczecin, Poland*)

Sergey Gefter (*V. N. Karazin Kharkiv National University, Ukraine*)

Anna Goncharuk (*V. N. Karazin Kharkiv National University, Ukraine*)

Graciela Adriana González (*Universidad de Buenos Aires, Argentina*)

Vyacheslav Gordevskyy (*V. N. Karazin Kharkiv National University, Ukraine*)

Natalya Hrudkina (*Technical University «Metinvest Polytechnik», Metinvest Holding LLC, Zaporizhzhya, Ukraine*)

Oleksii Hukalov (*B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, Kharkiv, Ukraine*)

Svetlana Ignatovich (*V. N. Karazin Kharkiv National University, Ukraine*)

Kateryna Khalina (*B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, Kharkiv, Ukraine*)

Aleksandr Kholkin (*Pryazovskyi State Technical University, Dnipro, Ukraine*)

Irina Kolupaieva (*Kharkiv National University of Radio Electronics, Ukraine*)

Valerii Korobov (*V. N. Karazin Kharkiv National University, Ukraine*)

Marcin Korzeń (*West Pomeranian University of Technology in Szczecin, Poland*)

Le Van Ngoc (*Institute of Post and Telecommunication Technology, Hanoi, Vietnam*)

Alexander Makarov (*V. N. Karazin Kharkiv National University, Ukraine*)

Piotr Mormul (*University of Warsaw, Poland*)

Efrain Cruz Mullisaca (*Universidad Mayor de San Andrés, La Paz-Bolivia*)

Bartosz Muszyński (*University of Szczecin, Poland*)

Olga Nesmelova (*Institute of Applied Mathematics and Mechanics, National Academy of Sciences of Ukraine, Sloviansk, Ukraine*)

Igor Nevliudov (*Kharkiv National University of Radio Electronics, Ukraine*)

Nguyen Khoa Son (*Vietnam Academy of Science and Technology, Hanoi, Vietnam*)

Verónica Estela Pastor (*Universidad de Buenos Aires, Argentina*)

Aleksey Piven' (*V. N. Karazin Kharkiv National University, Ukraine*)

Piotr Polak (*West Pomeranian University of Technology in Szczecin, Poland*)

Mykyta Popov (*Donbass State Pedagogical University, Sloviansk, Ukraine*)

Sergii Poslavskyi (*V. N. Karazin Kharkiv National University, Ukraine*)

Rabah Rabah (*IRCCyN – Ecole des Mines de Nantes, France*)

Tetiana Revina (*V. N. Karazin Kharkiv National University, Ukraine*)

Alexander Rezounenko (*V. N. Karazin Kharkiv National University, Ukraine*)

Yurii Romashov (*V. N. Karazin Kharkiv National University; Kharkiv National University of Radio Electronics, Ukraine*)

Daryna Shevchuk (*V. N. Karazin Kharkiv National University, Ukraine*)

Kateryna Shevtsova (*Donbass State Pedagogical University, Sloviansk, Ukraine*)

Grigorij Sklyar (*West Pomeranian University of Technology in Szczecin, Poland*)

Jekatierina Sklyar (*West Pomeranian University of Technology in Szczecin, Poland*)

Mykyta Sobur (*V. N. Karazin Kharkiv National University, Ukraine*)

Kateryna Stiepanova (*V. N. Karazin Kharkiv National University, Ukraine*)

Oleh Vozniak (*V. N. Karazin Kharkiv National University, Ukraine*)

Bartosz Wasilewski (*University of Szczecin, Poland*)

Jaroslaw Wozniak (*West Pomeranian University of Technology in Szczecin, Poland*)

Alexander Zuyev (*Institute of Applied Mathematics and Mechanics, National Academy of Sciences of Ukraine, Sloviansk, Ukraine; Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*)