Mathematical modeling of water management on urban territories: nonlinear dynamics, stability and controllability

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Introduction. Mathematical modeling of the heat, mass and biomass transfer in ecosystems is based on the ODEs for different compartments like atmosphere, surface water (SW), groundwater (GW), soils, used water (UW), saved water (SW), controlled water (CW), etc [1].

Recently, global climate change produced fast heat waves and gradual temperature rise, acceleration of the ice melting that accompanied by catastrophic events like tornado, flooding, draught, lack of food and drinking water [2]. Slow inevitable changes have been detected in the air water and soil pollutions, increasing surface water salinity, technical and drinking water quality, food shortages, level and frequency of the observed catastrophic events. All the changes have a negative impact on human, animal, plant and microbiome health, which leads to long-term nonlinear effects of primary factors that act in various combinations as synergistic ones, so that the mathematical models of recent years are aimed at solving local problems of drinking water, air quality, soil pollution, as well as future of the life at the planetary scale.

Since >50% of population live in the cities, the water management on urban territories based on data analysis and mathematical modeling is important to predict possible future risks [3]. Since 2013, Kharkiv region together with some other regions has high drought risk (Fig.1). In this study the water management model is studied on the datasets of city Kharkiv.

Fig.1. Drought risk on the regions of the Ukraine [www.wri.org].

Data analyses and mathematical modeling. The geophysical, water, air and soil pollution data on the territory of Kharkiv city and Kharkiv region have been studied, and the statistical regularities between the climate, weather, hydrological and ecological data have been obtained (Fig.2). For some of the pollutants the contamination level exceeds the maximum accepted values (red line in Fig.2).

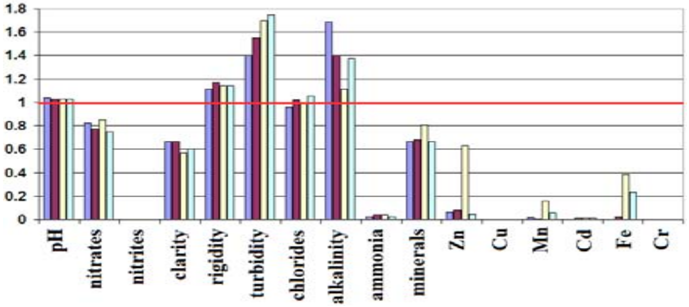

Fig.2. Contamination of Kharkov river by different pollutants at 4 locations (non-dimensional values).

The mathematical model of local water balance on a territory is based on the concept of 'compartments' (listed in Fig.3). Water inflow with precipitation, rivers and underground flows is balanced by the water outflow, evaporation and consumption [1-3].

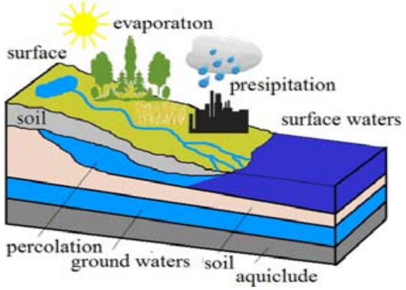

Fig.3. Compartmental model of the water balance system.

$$\frac{dW(t)}{dt} = \Sigma h(t) + G_W(t) + C_W(t) - U_W(N(t)) - Q(t) + R_W(N(t)), \quad (1)$$

$$\frac{dN(t)}{dt} = (b-d)N(t) + m, \quad (2)$$

$$\frac{dh(t)}{dt} = \frac{Hw + P_p c - Pwc}{\mu wc} - \frac{w+c}{\mu wc} h(t), \quad (3)$$

$$\frac{dM_j(t)}{dt} = \sigma_j(t) - k_j M_j(t) - \frac{a_j Q(t)}{1 + a_j Q(t)}, \quad (4)$$

where W(t) is the available water volume, N(t) is the local population number, h(t) is the ground water level, $M_j(t)$ are the mass concentrations of the pollutants j=1,2,…n (Fig.2).

Steady state and stability conditions for the system (1)-(4) have been studied with the model parameters on the territory of Kharkov city, and the criteria of its irreversible behavior have been derived. Different scenarios of the population growth and climate warming have been studied. It was shown, at some combinations of the parameters drastic water luck in the dynamical system could be obtained (Fig.4). The results will be used for water management planning at the governmental level at the conditions of further local climate changes.

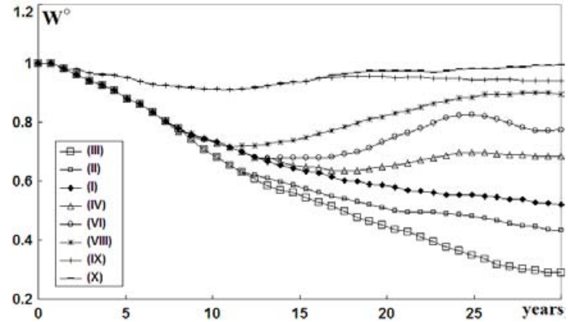

Fig.4. Prognosis of the non-dimensional water volume at different scenarios (I-X).

List of references

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