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Contents

On absolute stability of a class of time-varying switched nonlinear systems with delay

Nguyen Khoa Son, *Ha Noi, VietNam*

This paper is devoted to the stability analysis for a class of time-varying *switched nonlinear systems with delay* of the form

$$\dot{x}(t) = A_{\sigma(t)}(t)f(x(t)) + B_{\sigma(t)}(t)f(x_t), t \geq 0,\ \sigma \in \Sigma_+, \quad (1)$$

where $x_t = x(t+\cdot) \in C([-h,0],\mathbb{R}^n)$ with $h > 0$ being a given time-delay, Σ_+ is the set of *admissible switching signals* which are assumed to be piece-wise constant and right-side continuous functions $\sigma : [0,\infty) \to \underline{N} := \{1,2,\dots,N\}$ having on each bounded interval a finite number discontinuities $\tau_k, k = 1,2,\dots,$ known as the *switching instances*, $A_k(\cdot), B_k(\cdot) \in \mathbb{R}^{n\times n}$ are continuous matrix functions (see e.g. [1, 2, 3, 15] on the stability problems of switched systems). The nonlinear function $f : \mathbb{R}^n \to \mathbb{R}^n$ is assumed to be of the so-called Persidskii type, i.e. f is continuous, diagonal

$$f(x) = f(x_1,x_2,\dots,x_n) = (f_1(x_1), f_2(x_2),\dots,f_n(x_n)),$$

and satisfies the sector condition

$$0 < x_i f(x_i) \leq \beta_i x_i^2,\ \forall x_i \neq 0,\ \forall i \in \underline{n} \quad (2)$$

where $\beta_i > 0$ are given positive parameters. Such a nonlinear f is said to be *addmissble sector nonlinearity*. Then, cleary, $f(0) = 0$ and (1) admits the zero solution $x(t) \equiv 0, t \geq 0$. The zero solution of system (1) is said to be *absolute exponential stable* (or AES, for short) over Σ_+ if for any admissible switching signal $\sigma \in \Sigma_+$, any admissible nonlinearity f, and any initial condition $x_0 = \varphi, \varphi \in C([-h,0],\mathbb{R}^n)$ the corresponding solution $x(t) = x(t,\sigma,\varphi)$ of (1) satisfies

$$\|x(t)\| \leq Me^{-\alpha t}\|\varphi\|,\ \forall t \geq 0,$$

where $M > 0, \alpha > 0$ are certain real numbers. The reader is referred to, e.g. [6, 7, 8, 10, 14, 17] for the problems on absolute stability with sector nonlinearities. We are also inerested in a subclass $\Sigma_{\tau_a} \subset \Sigma_+$, consisting of switching signals $\sigma \in \Sigma_+$ having *average dwell time* (or ADT, for short) $\tau_a > 0$ which means that, for any $t > 0$, the number $N_\sigma(0,t)$ of disontinuities of σ on the interval $(0,t]$ satisfies

$$N_\sigma(0,t) \leq \frac{t}{\tau_a}, \quad (3)$$

(see,e.g. [1, 4]). Finally, for each matrix $P \in \mathbb{R}^{n\times n}$ we define a *Metzler matrix* $\bar{P}$ by setting $\bar{p}_{ii} = p_{ii},\ \bar{p}_{ij} = |p_{ij}|$, for $i, j = 1, ..., n, j \neq i$. The properties of Metzler matrices can be found in many papers concerning with *positive systems* , e.g. [5]. The main result of this paper is the following

Theorem 1 *Assume that there exist n-dimensional strictly positive vectors $\xi_k := (\xi_{k,1}, \xi_{k,2}, \dots, \xi_{k,n})^\top, \xi_{k,i} > 0, \forall k, i$, and a real number $\alpha > 0$ such that*

$$\big(\bar{A}_k(t) + e^{\alpha h}|B_k(t)|\big)\ D_\beta \xi_k \ll -\alpha\xi_k, \forall t \geq 0, \forall k \in \underline{N}, \tag{4}$$

where D_β is a diagonal matrix defined as $D_\beta = \mathrm{diag}(\beta_1, \beta_2, \dots, \beta_n)$. Then the zero solution of the system (1) is AES over the set Σ_{τ_a} of switching signals with ADT τ_a satisfying

$$\tau_a > \tau_* := \frac{\ln\ \gamma}{\alpha}, \tag{5}$$

where

$$\gamma := \max\Big\{\frac{\xi_{k,i}}{\xi_{l,i}} : k, l \in \underline{N}, i \in \underline{n}\Big\}. \tag{6}$$

Moreover, if there exists a strictly positive vector ξ such that (4) holds for $\xi_k = \xi, k \in \underline{N}$, then the zero solution of (1) is AES over the set of switching signals Σ_+.

The proof of the above theorem is based on the comparison principle, that is quite different from the traditional approach of all presvious works (see, e.g. [11, 13, 14]) where the method of Lyapunov-Krasovski functionals play a central role. In particular, if the system (1) is *positive* and time-invariant, i.e. $A_k(t) \equiv A_k = \bar{A}_k,\ B_k(t) \equiv B_k \geq 0, \forall k \in \underline{N}$ then Theorem 1 implies the following easily checkable criterion for AES.

Theorem 2 *Assume that switched system (1) is positive and time-invariant. Assume that there exist strictly positive n-dimensional vectors $\xi_k := (\xi_{k,1}, \xi_{k,2}, \dots, \xi_{k,n})^\top \gg 0$ such that*

$$(A_k + B_k)\xi_k \ll 0,\ \forall k \in \underline{N}, \tag{7}$$

Then the zero solution of the system is AES over the set Σ_{τ_a} of switching signals with ADT τ_a satisfying (5) where γ is defined by (6) and $\alpha = \min_{i\in\underline{n}, k\in\underline{N}} \alpha_{i,k}$, with $\alpha_{k,i}$ being the solutions of the equations, with $i \in \underline{n},\ k \in \underline{N}$,

$$\sum_{j=1}^{n} \big(a_{k,ij} + e^{\alpha h} b_{k,ij} + \alpha\beta_i^{-1}\big)\xi_{k,j} = 0. \tag{8}$$

Moreover, if there exists a strictly positive vector ξ *such that (7) holds for* $\xi_k = \xi, k \in \underline{N}$*, then the zero solution of (1) is AES over the set of switching signals* Σ_+*.*

The above theorems cover or improve a number of known sufficient conditions of absolute stability obtained by other authors (e.g. [11, 13, 14, 17, 18]), where only the case $\xi_k = \xi,\ k \in \underline{N}$ was considered.

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Controllability problems for wheeled electromechanical robotic platforms taking into account motion's smoothness restrictions

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The controllability problems are widely researched for the robotic wheeled electromechanical platforms, but considering of the motions' smoothness requirements is actually not researched now. At the same time, such requirements are significantly important for dangerous and delicate cargoes transportation for example under horizontal transportation of the nuclear fuel assemblies inside enterprises [1].

Although the linear models can represent only the limited cases of the motions, but these linear models are really suitable for considering the important controllability problems needed for engineering applications. So, the linear mathematical model of the wheeled electromechanical platform can be generally represented using the conventional denotation as follows:

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}, \quad \mathbf{x}(t_0) = \mathbf{x}_0. \tag{1}$$

The naturally needed changes in the motion of the platform during its operation require building the control signals providing such changes, so we have the controllability problem:

$$\mathbf{u}(t): \quad \mathbf{x}(t_1) = \mathbf{x}_1, \quad t_1 - t_0 \to \mathtt{min}. \tag{2}$$

The motion's smoothness restrictions [2] allows only the controls providing the limited acceleration derivative:

$$\|\mathbf{C} \cdot \dddot{\mathbf{x}}\| \leq a_{per}, \quad a_{per} > 0. \tag{3}$$

Exactly the indirect restriction (3) of the control makes difficulties in considering the controllability problem (1)-(3).

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Homogeneous approximations for control systems with output

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Let a series $S = \sum_I c(\eta_I)\eta_I$ be given, where η_I are iterated integrals, I denotes a multi-index, $I = (i_1 \dots i_k)$, $1 \le i_1, \dots, i_k \le m$, and $c(\eta_I)$ are scalar coefficients. It is called realizable if there exists an analytic control system of the form

$$\dot{x} = \sum_{i=1}^{m} X_i(x)u_i, \quad y = h(x),$$

such that its output $y = h(x)$ for trajectories starting at the origin, $x(0) = 0$, is represented by the series S, i.e., $y(t) = S(t, u) = \sum c(\eta_I)\eta_I(t, u)$. It is well known that the series is realizable if and only if its Lie rank is finite; in this case the Lie rank equals the minimal possible dimension of the realizing system [1].

We are interested in a homogeneous approximation problem for such series. We apply a free algebraic technique, which was developed to study homogeneous approximations for systems without output [2]. In particular, we generalize the concept of a core Lie subalgebra [3]. In the talk we propose a definition of the core Lie subalgebra for such series and describe its connection with the core Lie subalgebra of the realizing system. Namely, we present the following result.

Theorem *(i) The core Lie subalgebra of one-dimensional realizable series coincides with the core Lie subalgebra of its realization.*
(ii) Any graded Lie subalgebra of codimension n is a core Lie subalgebra for some one-dimensional series of Lie rank n.

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On stabilizability and controllability of triangular systems in the singular case

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Consider the class of nonlinear triangular systems

$$\begin{cases} \dot{x}_1 = f_1(u, x_1, \ldots, x_n), \\ \dot{x}_i = f_i(x_{i-1}, \ldots, x_n), \quad i = 2, \ldots, n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control. Triangular systems were introduced by V.I. Korobov in 1973 [1] where it is shown that system (1) is feedback linearizable if there exists a constant a such that $\frac{\partial f_i}{\partial x_{i-1}} \geq a > 0$ for every $x_{i-1} \ldots x_n$ $(x_0 = u)$.

Our main concern is the singular case in which $\frac{\partial f_n}{\partial x_{n-1}} = 0$ for $x = 0$ and system (1) is no longer linearizable. We find constructive solutions to the controllability and stabilizability problems for system (1) mapping it to the simpler nonlinear system under main requirement that $\left| \frac{\partial f_n^{\frac{1}{2k+1}}}{\partial x_{n-1}} \right| \geq a > 0$ for any x_{n-1}, x_n, where $a > 0$ is a constant, $k \in \mathbb{N}$. To this end, we construct the change of variables $z = F(x)$ and introduce the new control $v = G(x, u)$ transforming system (1) to the form

$$\begin{cases} \dot{z}_1 = v, \\ \dot{z}_i = z_{i-1}, \quad i = 2, \ldots, n-1, \\ \dot{z}_n = z_{n-1}^{2k+1}. \end{cases} \tag{2}$$

System (2) is inherently nonlinear system that has uncontrollable first approximation and can not be mapped to linear system. Constructive stabilizability and 0–controllability of system (2) are studied in [2],[3]. We propose a new way to construct a control $u = u(t, T, x_0, x_1)$ steering corresponding closed-loop system from any initial point x_0 to arbitrarily given final point x_1 at any finite time $T > 0$. We also construct a new class of stabilizing controls applying stability of cascade systems.

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Linear Noetherian difference-algebraic boundary value problems

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We investigate the problem of finding bounded solutions [1, 2, 3]

$$z(k) \in \mathbb{R}^n, \quad k \in \Omega := \{0,\ 1,\ 2,\ \dots,\ \omega\}$$

of linear Noetherian $(n \neq \upsilon)$ boundary value problem for a system of linear difference-algebraic equations

$$A(k)z(k+1) = B(k)z(k) + f(k), \quad \ell z(\cdot) = \alpha, \quad \alpha \in \mathbb{R}^{\upsilon}; \tag{1}$$

here $A(k),\ B(k) \in \mathbb{R}^{m\times n}$ are bounded matrices and $f(k)$ are real bounded column vectors,

$$\ell z(\cdot) : \mathbb{R}^n \to \mathbb{R}^{\upsilon}$$

is a linear bounded vector functional defined on a space of bounded functions. We assume that the matrix $A(k)$ is, generally speaking, rectangular: $m = n$. It can be square, but singular. The problem of finding bounded solutions $z(k)$ of a boundary value problem for a linear non-degenerate [2]

$$\det B(k) \neq 0, \quad k \in \Omega$$

system of first-order difference equations

$$z(k+1) = B(k)z(k) + f(k), \quad \ell z(\cdot) = \alpha \in \mathbb{R}^{\upsilon}$$

was solved by A.A. Boichuk [2]. Thus, the boundary value problem (1) is a generalization of the problem solved by A.A. Boichuk. We investigate the problem of finding bounded solutions linear Noetherian boundary value problem for a system of linear difference-algebraic equations (1) in case $1 \leq \operatorname{rank}\, A(k) = \sigma_0,\ k \in \Omega$. We construct necessary and sufficient conditions for the existence of solution of linear boundary value problem for a system of difference-algebraic equations in the critical and noncritical case [3].

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On of solving nonlinear integral-differential boundary value problems by the of Newton-Kantorovich method

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We are investigating the problem of constructing a solution

$$y(t) \in \mathbb{D}^2[a;b], \quad y'(t) \in \mathbb{L}^2[a;b]$$

nonlinear Noether $(n \neq p)$ integral-differential system

$$y'(t) = A(t)y(t) + \Phi(t) \int_a^b F(y(s), y'(s), s)\, ds + f(t), \tag{1}$$

that satisfy the boundary condition [1, 2, 3]

$$\ell y(\cdot) = \alpha, \quad \alpha \in \mathbb{R}^p. \tag{2}$$

We seek a solution of the Noetherian boundary value problem (1), (2) in a small neighborhood of solution

$$y_0(t) \in \mathbb{D}^2[a;b], \quad y_0'(t) \in \mathbb{L}^2[a;b]$$

of the generating problem

$$y_0'(t) = A(t)y_0(t) + f(t), \quad \ell y_0(\cdot) = \alpha. \tag{3}$$

Here

$$A(t) \in \mathbb{L}^2_{n\times n}[a;b] := \mathbb{L}^2[a;b] \otimes \mathbb{R}^{n\times n}, \quad \Phi(t) \in \mathbb{L}^2_{n\times m}[a;b], \ f(t) \in \mathbb{L}^2[a;b];$$

$\ell y(\cdot) \ : \ \mathbb{D}^2[a;b] \to \mathbb{R}^p$ —linear bounded vector functional defined in space $\mathbb{D}^2[a;b]$ n-dimensional absolutely continuous on a segment $[a,b]$ functions. Nonlinear vector-function $F(y(t), y'(t), t)$ twice continuously differentiable in the small neighborhood of the solution $y_0(t)$ generating boundary value problem (3), twice continuously differentiable with respect to $y_0'(t)$, and continuous in the independent variable t on the segment $[a,b]$.

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Nonlinear degenerate differential-algebraic boundary-value problems

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We will study the problem of construction of solutions

$$z(t,\varepsilon):\ z(\cdot,\varepsilon)\in\mathbb{C}^1[a,b],\ z(t,\cdot)\in\mathbb{C}[0,\varepsilon_0]$$

of the nonlinear differential-algebraic boundary-value problem [1, 2]

$$A(t)z'(t,\varepsilon)=B(t)z(t,\varepsilon)+f(t)+\varepsilon\, Z(z,t,\varepsilon),\ \ell z(\cdot,\varepsilon)=\alpha+\varepsilon\, J(z(\cdot,\varepsilon),\varepsilon). \qquad (1)$$

We will seek the solutions of the boundary-value problem (1) in a small neighborhood of a solution $z_0(t)\in\mathbb{C}^1[a,b]$ of the generating Noetherian $(n=q)$ boundary-value problem [3]

$$A(t)z_0'(t)=B(t)z_0(t)+f(t),\quad \ell z_0(\cdot)=\alpha. \qquad (2)$$

Here, $A(t),\ B(t)\in\mathbb{C}_{m\times n}[a,b]$ are continuous matrices, $f(t)\in\mathbb{C}[a,b]$ is a continuous vector; $Z(z,t,\varepsilon)$ is a nonlinear function which is continuously differentiable with respect to the unknown $z(t,\varepsilon)$ in a small neighborhood of a solution of the generating problem, continuous in $t\in[a,b]$, and continuous in a small parameter; $\ell z(\cdot,\varepsilon)$ and $J(z(\cdot,\varepsilon),\varepsilon)$ are, respectively, a linear and nonlinear vector functionals, $\ell z(\cdot,\varepsilon),\ J(z(\cdot,h,\varepsilon),\varepsilon):\mathbb{C}[a,b(\varepsilon)]\to\mathbb{R}^q$, Moreover, the second functional is continuously differentiable with respect to the unknown $z(t,\varepsilon)$ and continuous in the small parameter ε in a small neighborhood of a solution of the generating problem (2) and on the segment $[0,\varepsilon_0]$. The nonlinear differential-algebraic boundary-value problem (1) is a generalization of numerous statements of nonlinear boundary-value problems [1]. We will study the case of degeneration [3] of the generating boundary-value problem (2), namely: $P_{A^*}(t)\neq 0$; here, $P_{A^*}(t)$ is the orthoprojector [1]: $P_{A^*}(t):\mathbb{R}^m\to\mathbb{N}(A^*(t))$. Generally speaking, the degenerate system (2) is not solvable relative to the derivative. The necessary and sufficient conditions of solvability of nonlinear differential-algebraic boundary-value problem (1) and a convergent iterative scheme of construction of approximations to their solutions are found.

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Construction of invariant relations in the problem of estimating the speed of oscillations of oscillatory networks on incomplete information

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The problem of studying the collective behavior of multiscale dynamic processes is of fundamental importance for understanding the basic laws of synchronous dynamics with oscillations. This problem allows practical implementation in many problems of biology and mechanics, described using cyclic processes [1]. The problem of velocities determination for interconnected of systems described by the Lienar equations by known data is considered as observation problem in work. A new method – a synthesis of invariant relations [2] is used to design nonlinear observer. The method allows us to represent unknowns as a function of known quantities. The scheme of the construction of invariant relations consists in the expansion of the original dynamical system by equations of some controlled subsystem (integrator). Control in the additional system is used for the synthesis of some relations that are invariant for the extended system and have the attraction property for all of its trajectories. Such relations are considered in observation problems as additional equations for unknown state vector of initial oscillators ensemble. To design the observer, first we introduce a observer for unique Van der Pole oscillator and prove its exponential convergence. This observer is then extended on several coupled Van der Pole oscillators. The performance of the proposed method is investigated by numerical simulations [3].

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Mathematical model of solid fuel tablet combustion

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A one-dimensional math model of stationary solid fuel tablet combustion is studied. A state of constant flame intensity means a constant speed of the sublimation front in the tablet. The model takes into account the processes of heat transfer and the seepage of sublimation products through the pores. These processes are described by a system of partial differential equations with certain boundary conditions. The problem is a generalization of the classical Stefan problem for the case of a gas phase moving through a porous medium. We are interested in minimizing the amount of unburned tablet residues under certain conditions, for example, while limiting the possible porosity of the tablet.

The first and second kind matrix polynomials associated with the matrix Hamburger moment problem

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Let an infinite sequence $(s_l)_{l=0}^{\infty}$ of matrices from $\mathbb{C}^{m\times m}$ be given and for all $j \geq 0$ the block Hankel matrices $H_j = \left(s_{l+k}\right)_{l,\,k=0}^{j}$ are positive. We associate the matrix Hamburger moment problem with the matrix sequence $(s_l)_{l=0}^{\infty}$. This means, we want to describe all Hermitian $m \times m$ matrix measures σ such that

$$s_l = \int_{\mathbb{R}} t^l \sigma(dt), \qquad l \geq 0. \tag{1}$$

Using the first and second kind matrix polynomials P_j and Q_j (see [1]), we construct the infinite matrix columns

$$\pi(z) = \mathrm{col}\,(P_0(z), P_1(z), P_2(z), \ldots), \quad \xi(z) = \mathrm{col}\,(Q_0(z), Q_1(z), Q_2(z), \ldots).$$

We consider infinite matrix column vectors $V = \mathrm{col}\,(V_0, V_1, V_2, \dots),\ V_j \in \mathbb{C}^{m\times m}$. Denote by $\ell^2(\mathbb{C}^{m\times m})$ the set of all matricx columns V for which the matrix series $\sum_{j=0}^{\infty} V_j^* V_j$ converges. The following theorem is the main result of this talk (see [1]).

Theorem *Let the matrix Hamburger moment problem (1) be given. Then the following statements (1)-(5) are equivalent:*

(1) The moment problem (1) is completely indeterminate (see [1]).

(2) For some point $z_0 \in \mathbb{C} \setminus \mathbb{R}$ the column $\pi(z_0)$ belongs to $\ell^2(\mathbb{C}^{m\times m})$.

(3) For some point $z_0 \in \mathbb{C} \setminus \mathbb{R}$ the column $\xi(z_0)$ belongs to $\ell^2(\mathbb{C}^{m\times m})$.

(4) For some $x_0 \in \mathbb{R}$ both columns $\pi(x_0)$ and $\xi(x_0)$ belong to $\ell^2(\mathbb{C}^{m\times m})$.

(5) For all $z \in \mathbb{C}$ both columns $\pi(z)$ and $\xi(z)$ belong to $\ell^2(\mathbb{C}^{m\times m})$.

Furthermore, the following statements hold true:

(6) If for some $x_0 \in \mathbb{R}$ and some non-null vector $\phi \in \mathbb{C}^m$ both infinite column vectors $\pi(x_0)\phi$ and $\frac{d\pi}{dx}(x_0)\phi$ belong to the Hilbert space $\ell^2(\mathbb{C}^m)$, then the matrix Hamburger moment problem (1) is not completely determinate.

(7) If for some $x_0 \in \mathbb{R}$ and some non-null vector $\phi \in \mathbb{C}^m$ both infinite column vectors $\xi(x_0)\phi$ and $\frac{d\xi}{dx}(x_0)\phi$ belong to the Hilbert space $\ell^2(\mathbb{C}^m)$, then the matrix Hamburger moment problem (1) is not completely determinate.

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On controllability problems for the heat equation with variable coefficients on a half-axis

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We consider the following control system

$$w_t = \frac{1}{\rho}\left(kw_x\right)_x + \gamma w, \qquad x \in (0, +\infty),\ t \in (0, T), \tag{1}$$

$$w(0, \cdot) = u, \qquad t \in (0, T), \tag{2}$$

$$w(\cdot, 0) = w^0, \qquad x \in (0, +\infty). \tag{3}$$

Here $T > 0$ is a constant; $u \in L^\infty(0, T)$ is a control; ρ, k, γ, and w^0 are given functions. We also assume $\rho, k \in C^1[0, +\infty)$ are positive on $[0, +\infty)$, $(\rho k) \in C^2[0, +\infty)$, $(\rho k)'(0) = 0$, and $\sigma(x) = \int_0^x \sqrt{\rho(|\xi|)/k(|\xi|)}\, d\xi \to \pm\infty$ as $x \to \pm\infty$. In addition, we assume $Q(k, \rho) - \gamma \in L^\infty(0, +\infty) \bigcap C^1[0, +\infty)$ and $\sigma\sqrt{\rho/k}\,(Q(k, \rho) - \gamma) \in L^1(0, +\infty)$, where $Q(k, \rho) = \sqrt{k/\rho}\left(\sqrt{k/\rho}(k\rho)'/(4k\rho)\right)' + \left(\sqrt{k/\rho}(k\rho)'/(4k\rho)\right)^2$. We consider control system (1)–(3) in modified Sobolev spaces.

It is proved that each initial state of control system (1)–(3) is approximately controllable to any target state in a given time $T > 0$. In the case of constant coefficients $(\rho = k = 1, \gamma = 0)$, this result has been obtained earlier in [1]. If an initial state of the control system is null-controllable (i.e., if the target state is the origin), then the initial state is the origin . In the case of constant coefficients $(\rho = k = 1, \gamma = 0)$, this result has been obtained earlier in [1].

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Linear differential equations in the module of copolynomials

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Let K be an arbitrary commutative ring with identity, $K[x]$ is the ring of polynomials with coefficients in K and $K[x]'$ is a module of homomorphisms (K-linear mappings) from $K[x]$ to K. By a copolynomial over the ring K we mean an element of the module $K[x]'$. The derivative T' of a copolynomial T is defined in the same way as in the classical theory of generalized functions: $(T', p) = -(T, p')$, $p \in K[x]$. We have studied some linear ordinary differential equations and partial differential equations in the module $K[x]'$. We presents one of these results here.

Theorem *Let a be an invertible element of the ring K and $b, c \in K$. Then for any copolynomial $T \in K[x]'$ there exists a unique solution $w \in K[x]'$ to the equation*

$$cw'' + bw' + aw = T.$$

This solution has the form

$$w = \sum_{n=0}^{\infty} \sum_{j=0}^{\left[\frac{n}{2}\right]} (-1)^{n-j} C_{n-j}^{j} a^{j-n-1} b^{n-2j} c^{j} T^{(n)},$$

where the series converges in a natural weak topology on $K[x]'$.

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Linear differential equations in the ring of formal power series

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Let us consider the differential equation with real coefficients

$$a_m w^{(m)}(x) + a_{m-1} w^{(m-1)}(x) + \ldots + a_1 w'(x) + a_0 w(x) = f(x). \tag{1}$$

It is known (see [1, §5, 22.2]), that of the sum of the series

$$w(x) = \sum_{i=0}^{\infty} c_i f^{(i)}(x), \tag{2}$$

where c_i are the coefficients of

$$(a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0)^{-1} = c_0 + c_1 s + c_2 s^2 + \ldots$$

solves the non-homogeneous equation if some convergence conditions hold.

Let K be an integral domain, R is the quotient field of K, $a_0, \ldots, a_m \in K$ and $f(x) \in K[[x]]$. Consider a problem of finding the function $w(x) \in K[[x]]$, which satisfied the differential equation (1).

Theorem *Suppose $a_0 \neq 0$. If $f(x)$ is a polynomial from $K[x]$ then Equation (1) has a unique solution from $R[x]$, and the solution has the form (2).*

If $f(x)$ is a formal power series, but not a polynomial, the sum (2) is not a well-defined formal power series. By previous theorem, the equation in this case has no polynomial solution, but it still can has a solution from $K[[x]]$. The following theorem gives us a sufficient conditions for existing and uniqueness of the solution from $K[[x]]$.

Theorem *Suppose K is a valuation ring of a field F with a non-Archimedean valuation $|\cdot|$. If $|a_0| = 1$ and $|a_i| < 1$ for any $1 \leq i \leq m$, then the series (2) is well-defined and it is a unique solution of the equation (1) from $K[[x]]$.*

The research was supported by the National Research Foundation of Ukraine funded by Ukrainian State budget in frames of project 2020.02/0096 "Operators in infinite-dimensional spaces: the interplay between geometry, algebra and topology".

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Application of energy method for the analysis of technological modes the processes of precise stamping by extrusion

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The study is aimed at solving an scientific and technical problem of improving the efficiency of plastic deformation processes on the basis of the development of analysis methods of precise volumetric stamping processes by extrusion [1]. Research consists in the development of a complex of kinematic modules to simulate the flow of metal taking into account the design features of the tool in the form of elements of transition sections. The possibilities of the energy method are developed due to the designing of kinematic modules of trapezoidal and triangular shapes, recommendations are given for the rationality of their usage [2]. Restrictions on the shapes of curves describing the boundaries of kinematic modules of triangular and trapezoidal shapes and configurations of adjacent modules are revealed. For elimination the problem of the impossibility of using a quarter of a circle as an boundary for a kinematic trapezoidal module, it is proposed to use an approximate function whose deviations in the length of the arc do not exceed 0.8 percent [3]. It significantly expands the possibilities of using the energy method for modeling processes with a tool configuration in the form of edges and rounding. Technological recommendations for design of the processes of precise stamping by extrusion are developed in compliance with the main stages of process development on the basis of the classification of kinematic modules. Technical solutions and methods, the developed modules of the software implementation of the processes were transferred to a number of enterprises and are used in the research works of the Donbass State Engineering Academy.

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Pullback and forward dynamics of nonautonomous difference equations: Basic constructions

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In order to comprehensively capture the long-term behaviour of nonautonomous difference equations, both pullback and forward attractors as well as forward limit sets are constructed for general infinite-dimensional nonautonomous dynamical systems in discrete time and complete metric spaces. While the theory of pullback attractors is well-established, the present novel approach is needed in order to understand their future behaviour.

More complicated equations and the behaviour of their attractors under spatial discretisation will be tackled in future papers.

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The identification of the rules processes' regulation during restoration (regeneration) of dynamic homeostasis by methods of Adaptive Dynamic Programming

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Liver regeneration is one of the most captivating phenomena in medicine.The identification of the main dependencies that determine the strategy of liver regeneration is one of the main problems in the regenerative medicine. The mathematical model that qualitatively describes the processes of liver regeneration in explicit dependence on the control parameters is developed [1]. This model represents the processes of replication, polyploidization, binuclear cells, hyperplasia, effects of toxic factors, apoptosis, cell death and the effects of secondary toxicity.

The dynamics of populations of liver cells is given by the equation:
$\overline{x}(t+1) = f(\overline{x}(t), \tau(t), \overline{\lambda}(t))$, where $\overline{x}(t)$ - types of functional liver cells at moment t, $\tau(t)$ - given function of external toxicity, $\overline{\lambda}(t)$ - control parameters.

The generalized liver function index is $\Phi(t) = \sum_{i=0}^{m} c_i(x_i(t), \tau(t))$, where c_i - own index of functionality for cell type $x_i(t)$. $0 \leq \Phi(t) \leq 1$, where 0 denotes a dead organism and 1 is the most functional organism. Therefore the change of the organism's functional state is described by the equation: $\widetilde{\Phi}(t+1) = \Psi(\tau(t), \Phi(t))$.

Numerical calculations confirm that the mathematical model corresponds to biological processes for different strategies of liver regeneration[2]. For the feedback control of liver regeneration system it is proposed to use a family of techniques known as Approximate or Adaptive Dynamic Programming (also known as Neurodynamic Programming)[3].

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The large-time asymptotics for the modified Camassa–Holm equation on a non-zero background

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We consider the initial value (Cauchy) problem for the modified Camassa–Holm (mCH) equation:

$$m_t + \left((u^2 - u_x^2)m\right)_x = 0, \quad m := u - u_{xx}, \quad t > 0; \quad -\infty < x < +\infty,$$

$$u(x,0) = u_0(x), \quad -\infty < x < +\infty,$$

assuming that $u_0(x) \to 1$ as $x \to \pm\infty$ and that the time evolution preserves this behavior: $u(x,t) \to 1$ as $x \to \pm\infty$ for all $t > 0$.

In [1], we have developed the Riemann–Hilbert formalism for this problem, which allowed us to represent the solution of the Cauchy problem in terms of the solution of an associated Riemann–Hilbert factorization problem. The present work (see also [2]) aims at the large-time asymptotic analysis of solution of the Cauchy problem mentioned above by the nonlinear steepest descent method, based on the developed Riemann–Hilbert formalism. Particularly, we show that in the solitonless case, the asymptotics in two sectors of the (x,t) half-plane, $1 < \zeta := \frac{x}{t} < 3$ and $\frac{3}{4} < \zeta < 1$, where the deviation from the background value is nontrivial, is as follows:

- $1 < \zeta < 3$: $u(x,t) = 1 + \frac{C_1}{\sqrt{t}} \cos\{C_2 t + C_3 \ln t + C_4\} + o(t^{-1/2})$;
- $\frac{3}{4} < \zeta < 1$: $u(x,t) = 1 + \sum_{j=0,1} \frac{C_1^{(j)}}{\sqrt{t}} \cos\left\{C_2^{(j)} t + C_3^{(j)} \ln t + C_4^{(j)}\right\} + o(t^{-1/2})$,

where C_i, $C_i^{(j)}$ are functions of ζ specified in terms of the scattering data, which in turn are uniquely specified by the initial data for the Cauchy problem.

In the remaining sectors $\frac{x}{t} > 3$ and $\frac{x}{t} < \frac{3}{4}$, $u(x,t)$ decays rapidly (exponentially fast) to 1.

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Operator-Theoretic Proof of Arnold's Theorems of Alternation and Non-oscillation for Differential Equations of Even Order with Operator Coefficients

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For differential equations with operator coefficients in the joint works by F.S. Rofe - Beketov and the author studied the interplay between spectral and oscillatory properties of such problems [1] - [3]. The interesting topological interpretation of Sturm's theorems for the matrix Sturm - Liouville equation with real coefficients was considered by V.I. Arnold [4].

In the present work, we consider self- adjoint differential equations of an arbitrary even order with operator coefficients from $B\left(H\right)$ (H - separable Hilbert space)

$$l\left[y\right]=\sum_{k=1}^{n}\left(-1\right)^{k}\left\{\left(p_{k}y^{(k)}\right)^{(k)}-\frac{i}{2}\left[\left(q_{k}y^{(k)}\right)^{(k-1)}+\left(q_{k}^{*}y^{(k-1)}\right)^{(k)}\right]\right\}+$$

$$+p_{0}\left(x\right)y=\lambda W\left(x\right)y,\qquad a\leqslant x\leqslant b<\infty,$$

where the coefficients $p_{k}\left(x\right)=p_{k}^{*}\left(x\right),\, q_{k}\left(x\right)$ depend continuously in the uniform sense on x together with their derivatives up to the order k inclusively, and $p_{n}\left(x\right)>>0,\;\; W\left(x\right)=W^{*}\left(x\right)>>0,\;\; y\left(x\right)-$the vector is functions with values in H. An oscillation theorem is proved for such equations in the case of a boundary condition of the general form. Using this theorem, a generalization of Arnold's alternation theorem is obtained. It is shown that Arnold's non - oscillation theorem for an equation of arbitrary even order in the corrected form follows from [2] - [3]. This is proved by the operator-theoretic method. A generalization and a correction of theorems of zeros for differential equations of an arbitrary even order with operator-valued coefficients were obtained. There, a variety of Arnold's theorem of comparison was established as well.

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Big Data analysis and mathematical modeling of Covid-19

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Statistical analysis of the time series on covid-19 available from the open datasets, and mathematical modeling of the complex nonlinear dynamics of the number of healthy $S(t)$, exposed $E(t)$, quarantined $Q(t)$, infected $I(t)$, active $A(t)$, vaccinated $V(t)$, recovered $R(t)$ and dead $D(t)$ individuals have been a subject for intense studies in 2020-2021 aimed at stability analysis, controllability and detailed prognosis of the pandemic in different countries [1, 2].

Here a brief review on the existing mathematical models in the form

$$\frac{dN_j}{dt} = F\left(N_1(t-\tau_1), N_2(t-\tau_2), ..., N_n(t-\tau_n)\right), \tag{1}$$

where $N_j(t), j = 1, 2, ...n$ are the measured values of $E(t), I(t), A(t), R(t), D(t)$, $\tau_j(t), j = 1, 2, ...n$ are the corresponding time delays, is given, including the most popular $S-I-R$, $S-I-R-S$, $S-E-I-R-S$, $S-E-I-Q-R$ models without/with time delay.

Statistical analyses of the time series on 3-4 pandemic 'waves' in the Ukraine and neighbor European countries have been carried out. It was shown, the main peaks in the daily measured $I(t)$ and $D(t)$ curves very well correlated for the second and third 'waves' when the time delay is subtracted. The 'nearest neighbor' method, and cross-correlation studies have been used for classification and cluster analysis of the covid19 dynamics in different countries.

It was shown, the time delay dynamics between $I(t)$ and $D(t)$ curves can be described by a corrected $S-E-I-R-S$ model without time delay. The model parameters in the $S-E-I-R-S$ model (1) have been estimated based on the literature published for different countries [1, 2]. It was shown, the criterion of stability of the ODE system (1) $\Xi < \xi^*$ corresponds to the reproduction rate coefficient which is important for the decision on local antiepidemic measures.

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Mathematical modeling of water management on urban territories: nonlinear dynamics, stability and controllability

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Mathematical modeling of the heat, mass and biomass transfer in ecosystems is based on the ODEs for different compartments like atmosphere, surface water (SW), groundwater (GW), soils, used water (UW), saved water (SW), controlled water (CW), etc [1]. Recently, global climate change produced fast heat waves and gradual temperature rise, acceleration of the ice melting that accompanied by catastrophic events like tornado, flooding, draught, lack of food and drinking water [2]. Since $> 50\%$ of population live in the cities, the water management on urban territories based on data analysis and mathematical modeling is important to predict possible future risks [3].

Here a brief review on the existing system dynamics models in the form

$$\frac{dW}{dt} = k_1 W_1(t) + k_2 W_2(t) + k_3 W_3(t) - k_4 W_4(t) - k_5 W_5(t), \tag{1}$$

where $\vec{W(t)} = (SW(t), GW(t), CW(t), UW(t), LW(t))$, $W(t)$ is the water contents, k_{1-5} are constant values known from local statistical reports, is given.

The geophysical, water, air and soil pollution data on the territory of Kharkiv city and Kharkiv region have been studied, and the statistical regularities between the climate, weather, hydrological and ecological data have been obtained. The computed values k_{1-5} have been used for adjustment of the model (1) to the local environmental conditions. Stability of the obtained system of ODEs for small excitations have been studied, and the criteria of its irreversible behavior have been derived. The results will be used for water management planning at the governmental level at the conditions of further local climate changes.

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The feedback synthesis for motion of a mass on an ideal spring

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Let us consider a material point attached to a spring sliding on a frictionless surface. We suppose that the control is attached to the material point:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -x_1 + rx_1 + u. \end{cases} \tag{1}$$

Here $t \geq 0,\ (x_1, x_2) \in Q \subset \mathbb{R}^2$ is a state, Q is a neighborhood of the origin, u is a scalar control (controllable engine power) satisfying the constraint $|u| \leq 1$, the value r is an *unknown constant* bounded perturbation which satisfies the preassigned constraint $|r| \leq \Delta < 1$. The approach presented in the talk is based on the Controllability Function method proposed by V.I. Korobov in 1979.

Theorem *Let $0 < \gamma < 1$ and the Controllability Function $\Theta = \Theta(x_1, x_2)$ be the unique positive solution to Eq.*

$$\begin{aligned} &\tfrac{2}{3}(2\Theta^4 - 2\Theta^2 + \cos(2\Theta) + 2\Theta\sin(2\Theta) - 1) = 4x_1^2(2\Theta^2 - \cos(2\Theta) + 1) + \\ &+ 8x_1x_2(2\Theta - \sin(2\Theta)) + 4x_2^2(2\Theta^2 + \cos(2\Theta) + 1). \end{aligned} \tag{2}$$

Let the solvability domain be the ellipsoid Q defined by $Q = \{(x_1, x_2) :\ \Theta(x_1, x_2) \leq c\}$. Let

$$\Delta = \frac{(1-\gamma)}{0.95 + 0.0625\sqrt{64c^2 + 96c + 87.84}}. \tag{3}$$

Then for all $|r| \leq \Delta$ in the ellipsoid Q the control given by

$$u(x_1, x_2) = \frac{2x_1\Theta(\sin(2\Theta) - 2\Theta) + 2x_2\Theta(-2\Theta^2 - \cos(2\Theta) + 1)}{2\Theta^4 - 2\Theta^2 + \cos(2\Theta) + 2\Theta\sin(2\Theta) - 1} \tag{4}$$

solves the local feedback synthesis for robust system (1). Moreover, the trajectory $x(t)$ of the closed-loop system, starting at an arbitrary initial point $x(0) = x_0 \in Q$ ends at the origin at some finite time (settling-time function) $T(x_0, r)$ satisfying the estimate $T(x_0, r) \leq \frac{\Theta(x_0)}{\gamma}$.

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The synthesis problem for LS to a non-equilibrium point

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We are going to construct a control $u(t)$ of the following type of system

$$\dot{x} = Ax + bu, \quad x \in \mathbb{R}^n, \ u \in \Omega \subset \mathbb{R}^m, \tag{1}$$

for which the trajectory starting at an arbitrary point $x_0 \in \mathbb{R}^n$ transfers into a given non-equilibrium point $x_T \in \mathbb{R}^n$ in a finite time $T = T(x_0, x_T)$. Assume that the condition $(b, Ab, \dots, A^{n-1}b) = n$ holds. If this condition is not satisfied, then it is impossible to solve problem of synthesis [1]. Since we are interested in the study of globally controlled systems, $\|u(t)\| \leq d$ for all $t \in [0, T]$ hold and eigenvalues of the matrix A have non-positive real parts. Let $u(t)$ be a control, which transfers x_0 to x_T in a finite time along the trajectory $x(t)$ of system (1) according to the Cauchy formula: $x(t) = e^{At}\Big(x_0 + \int\limits_0^t e^{-A\tau} bu(\tau) d\tau\Big)$. Let us denote $x(T) = x_T$, put $t = T$ which is the time of getting to the x_T:

$$x_0 - e^{-AT} x_T = -\int\limits_0^T e^{-A\tau} bu(\tau) d\tau. \tag{2}$$

If T were given, then the problem of getting to a stationary or even non-equilibrium point would be reduced to the problem of getting from a fixed point $(x_0 - e^{-AT} x_T)$ to zero. The difficulty is that we do not know T, and the left side of equality (2) depends on T. By virtue of identity (2), it is enough to find a trajectory that connects $(x_0 - e^{-AT} x_T)$ and the origin in a finite time T. Since we need to find this time T, it is also clear that

1) $x_0 - e^{-AT} x_T$ is not given;

2) x_T is not equilibrium point of the initial system.

Construction of the control $u(t)$, which transfers $(x_0 - e^{-AT} x_T)$ to (0; 0) in time T and satisfies the preassigned constraints, will be carried out in [2] by controllability function method [3].

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Kumpera-Ruiz algebras and related issues (21 years after a Trieste 2000 conference)

Piotr Mormul, *Warsaw, Poland*

In March 1998 H. Sussmann asked if all Goursat distributions were locally nilpotentizable – whether they locally possessed a basis of sections generating a nilpotent Lie algebra over the constants: $D = \mathrm{span}(X, Y)$ locally, $\mathrm{Lie}_{\mathbb{R}}(X, Y)$ – nilpotent. (A synonyme for local nilpotentizability is *feedback nilpotentization*.) Sussmann's question fell into the framework of his program formulated in the year 1993:

(...) A third important issue is that of fully exploiting the possibilities of feedback nilpotentization. This requires that one look for new classes of nilpotentizable systems, and also that one improve the existing nilpotentization results by making them as explicit as possible. (...)

During a conference in Trieste in June 2000 I answered Sussmann's question by YES. The emerging algebras were called *Kumpera-Ruiz*. Moreover, I gave effective recursive formulas for the nilpotency orders of the Kumpera-Ruiz algebras.

In contemporary language the local nilpotentizability is called *weak nilpotency*, as contrasted to *strong nilpotency* – a relatively new notion sprung into the existence only in the year 2000.

Relations between the weak and strong nilpotencies for Goursat distributions will be briefly outlined during my presentation, along with the remaining open problems.

Asymptotic analysis of classes of C_0-groups with generators having non-basis family of eigenvectors

Grigory Sklyar, *Szczecin, Poland*
Vitalii Marchenko, *Kharkiv, Ukraine*
Piotr Polak, *Szczecin, Poland*

The talk is devoted to asymptotic analysis of classes of C_0-groups constructed in 2017 by G. Sklyar and V. Marchenko in [1]. Unbounded generators of considered C_0-groups of fixed class $k \in \mathbb{N}$ have purely imaginary eigenvalues

$$\lambda_n = if(n), \quad n \in \mathbb{N},$$

where

$$\{f(n)\}_{n=1}^{\infty} \in \mathcal{S}_k = \left\{ \{f(n)\}_{n=1}^{\infty} \subset \mathbb{R} : \lim_{n\to\infty} f(n) = +\infty; \left\{ n^j \Delta^j f(n) \right\}_{n=1}^{\infty} \in \ell_\infty \text{ for } 1 \le j \le k \right\},$$

and corresponding complete minimal family of eigenvectors, which however does not form a Schauder basis. Following ideas of [3] and developing them we proved that under only one spectral condition, i.e. if $\exists$ a constant $K > 0$ such that $\forall n \in \mathbb{N}$ we have

$$n\,|\Delta f(n)| \ge K, \tag{1}$$

exact two-sided polynomial bounds for norms of corresponding C_0-groups hold. This means that C_0-groups grow as $|t|^k$, $t \to \pm\infty$. However we showed that these C_0-groups do not have any maximal asymptotics. This means that the fastest growing orbits do not exist. Note that the construction of these special classes of C_0-groups from [1] allowed to prove in [2] that the XYZ Theorem is sharp. The latter theorem (the XYZ Theorem) on the Riesz basis property for invariant subspaces of the generator of the C_0-group was obtained a decade ago by G.Q. Xu, S.P. Yung and H. Zwart in [4], [5].

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On the extension of Batty's theorem on the semigroup asymptotic stability

Grigory Sklyar, *Szczecin, Poland*
Piotr Polak, *Szczecin, Poland*
Bartosz Wasilewski, *Szczecin, Poland*

The well-known Batty's theorem [1] states that if a C_0-semigroup $T(t)$ is bounded and the spectrum of the generator A is contained in the open left-half plane of $\mathbb{C}$, then $\|T(t)A^{-1}\|$ tends to 0. This can be thought of as a particular case of a more general property that, for $\omega_0 > -\infty$ and $(\omega_0 + i\mathbb{R}) \cap \sigma(A) = \emptyset$ it holds $\|T(t)(A - \omega_0 I)^{-1}\|/\|T(t)\|$ tends to 0. We show that it is true for $\|T(t)\|$ regular enough, however we give examples [2] of unbounded semigroups, with the spectrum of the generator not contained in the open left-half plane of $\mathbb{C}$, with the above property. Moreover we give a more general sufficient condition for this property to hold, thus extending Batty's theorem.

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Linearizability problem and invariants of linear control systems with analytic matrices

Katerina Sklyar, *Szczecin, Poland*
Svetlana Ignatovich, *Kharkiv, Ukraine*

The problem of linearizability of control systems was a focus of attention of researches during several decades. The paper of V. I. Korobov [1] was the first work in which a class of systems was proposed that admit feedback linearization (triangular systems). Later, the idea was developed for more general classes of systems. An alternative approach was proposed by A. Krener, where the Lie bracket technique was applied, requiring C^∞-smoothness. This direction was intensively developed by many authors [2]-[4] under restrictive requirements for the smoothness of the system. Fundamentally new results were obtained in [5], where the linearizability problem for general affine systems of class C^1 was solved.

However, the study of the linearizability problem was mainly concerned with autonomous systems. The new step was taken in [6], namely, conditions of mappability of a nonlinear non-autonomous systems to linear non-autonomous systems with analytic matrices were obtained. An important role here is played by driftless systems of the form $\dot{x} = b(t)u$. It turns out that the vector function $\gamma(t) = K^{-1}(t)b^{(n)}(t)$, where $K = (b(t), \dot{b}(t), \ldots, b^{(n-1)}(t))$, is invariant w.r.t. changes of variables. In the talk, we discuss the following "realizability problem": which functions $\gamma_k(t)$ can be invariants of some linear driftless system. This problem is closely connected with the classical study of homogeneous linear ODE with meromorphic coefficients. We show how the answer helps in describing linearization conditions.

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Uniqueness of large solutions of semilinear elliptic equations with degenerate absorption

Yevgeniia Yevgenieva, *Sloviansk, Ukraine*

In bounded domain $\Omega \in \mathbb{R}^N$ with C^2–smooth boundary $\partial\Omega$ we consider the semilinear equation:

$$Lu + H(x)u^p := -\sum_{i,j=1}^{N} \left(a_{ij}(x)u_{x_i}\right)_{x_j} + H(x)u^p = 0 \quad \text{in } \Omega,\ p > 1, \tag{1}$$

where $C^{1,\lambda}$–smooth functions $a_{ij}(\cdot)$ satisfy the ellipticity condition

$$d_1|\xi|^2 \geqslant \sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geqslant d_0|\xi|^2 \quad \forall\, \xi \in \mathbb{R}^N,\ \forall\, x \in \overline{\Omega},\ d_1 < \infty,\ d_0 > 0,$$

and absorption potential $H(\cdot)$ satisfies

$$H(x) \geqslant h_\omega(d(x)) \quad \forall\, x \in \overline{\Omega}, \quad h_\omega(s) =: \exp\left(-\frac{\omega(s)}{s}\right) \quad \forall\, s \in (0, \rho_0). \tag{2}$$

Equation (1) with boundary condition

$$\lim_{d(x)\to 0} u(x) = \infty, \quad d(x) := dist(x, \partial\Omega),$$

is called *large solution* [1].

Theorem [2] *Let potential H satisfies estimate (2), where nondecreasing continuous function $\omega(\cdot)$ satisfies the technical condition:*

$$s^{\gamma_1} \leqslant \omega(s) < \omega_0 = const < \infty \quad s \in (0, \rho_0),\ 0 < \gamma_1 < 1$$

and the Dini condition

$$\int_0^c \frac{\omega(s)}{s} ds < \infty; \tag{3}$$

Then equation (1) admit only one large solution in mentioned domain Ω.

Remark *We conjecture that Dini condition (3) is also a necessary condition for the uniqueness of the large solution.*

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