V. N. Karazin Kharkiv National University
Department of Applied Mathematics
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PLENARY COMMUNICATIONS

The Green's operator of a generalized matrix differential-algebraic boundary value problem with pulse action

Sergey Chuiko, *Ukraine*

We set forth solvability conditions and construction of the generalized Green's operator for a linear matrix differential-algebraic boundary value problem with pulse action. Sufficient conditions of reducibility of a generalized matrix differential-algebraic operator to a conventional differential-algebraic equation with an unknown column vector are established. To solve a matrix differential-algebraic boundary value problem with pulse action, we employ a special solvability conditions and the construction of a general solution to a matrix Sylvester-type equation. The formula of a particular solution to the Sylvester-type equation can be used in stability theory [1]. The above solvability conditions and the construction of the generalized Green's operator of the Noetherian linear matrix differential-algebraic boundary value problem with pulse action generalize the conventional results for Noetherian boundary value problems for systems of ordinary differential equations with pulse action [2] as well as differential -algebraic systems of equations [3].

[1] Korobov V. I. and Bebiya M. O. Stabilization of some class of nonlinear systems those are uncontrollable in the first approximation// Dokl. NAN Ukrainy, 2014, no. 2,pp. 20-25.

[2] Boichuk A. A. and Samoilenko A. M., "Generalized Inverse Operators and Fredholm Boundary-Value Problems", VSP, Utrecht and Boston, 2004.

[3] Chuiko S. M. The Green's operator of a generalized matrix linear differential-algebraic boundary value problem// Siberian Mathematical Journal, 2015, no. 4, pp. 752-760.

Transformation operators and controllability problems for the wave equation with variable coefficients

Larissa Fardigola, *Kharkiv, Ukraine*

In the paper, novel transformation operators and special spaces of the Sobolev type are introduced and studied for the differential operator $\frac{1}{\rho(x)}\frac{d}{dx}\left(k(x)\frac{d}{dx}(\cdot)\right)$. Here $\rho, k \in C^1[0,a)$ are positive functions satisfying some additional restrictions. Controllability properties of the wave equation

$$z_{tt} = \frac{1}{\rho}\left(kz_x\right)_x + \gamma z, \qquad x \in (0,a),\ t > 0, \tag{1}$$

depend on the behaviour of $\sigma(x) = \int_0^x \sqrt{\rho(\xi)/k(\xi)}\,d\xi$. This equation is considered in the introduced spaces of Sobolev type. Denote $b = \lim_{x\to a}\sigma(x)$. By using the introduced transformation operators, it is proved that equation (1) replicates controllability properties of the wave equation

$$w_{tt} = w_{\xi\xi} - q^2 w, \qquad \xi \in (0,b),\ t > 0, \tag{2}$$

where $q \geq 0$ is the constant determined by ρ, k, and γ. We consider four cases: (i) $a = +\infty$ and $b = +\infty$; (ii) $a = +\infty$ and $b = d$; (iii) $a = l$ and $b = +\infty$; (iv) $a = l$ and $b = d$, where $l, d > 0$. Moreover, $q = 0$ in cases (ii) and (iv). Necessary and sufficient conditions for (approximate) L^∞-controllability of the wave equation with variable coefficients are obtained in the paper.

Normalization of homogeneous approximations under feedbacks

Svetlana Ignatovich, *Kharkiv, Ukraine*

We consider nonlinear control systems that are linear with respect to control,

$$\dot{x} = \sum_{i=1}^{m} X_i(x)u_i,$$

where vector fields X_i are assumed to be real analytic in a neighborhood of the origin in $\mathbb{R}^n$. We show how, in studying local properties of such systems, algebraic objects arise, namely, a free associative algebra, a free Lie algebra, formal power series of non-commuting variables, shuffle product. We discuss a homogeneous approximation problem — in a certain sense, an analogue of approximating of functions by the first terms of their Taylor series. These "free-algebraic" tools allow us to obtain a complete classification of homogeneous approximations: under the accessibility property, the set of all homogeneous approximations is in a one-to-one correspondence with the set of all graded Lie subalgebras of codimension n of the free Lie algebra generated by m elements.

Then we study a classification of homogeneous approximations up to linear changes of controls. We show that feedbacks correspond to linear transformations of basis in the free associative algebra. The problem is to describe all *growth vectors* such that all homogeneous approximations corresponding to them can be reduced to a finite number of *normal forms* by feedbacks. (Such growth vectors are called *normal.*) This problem is completely solved for $m = 2$.

Some qualitative properties of solutions to parabolic partial differential equations with state-dependent delays

Alexander Rezounenko, *Kharkiv, Ukraine*

We investigate a wide class of parabolic nonlinear evolution equations with discrete state-dependent delay. It is well-known that reaction terms with discrete state-dependent delay are more sensitive than the ones with distributed delays. It requires more attention when choosing between different phase spaces. We are interested in the well-posedness of the initial value problem in different spaces. Several possible choices of phase spaces are presented. We also study the existence of global and exponential attractors. The recently developed method of quasi-stability estimates (I.D. Chueshov and I. Lasiecka) allows us to prove that in some cases the attractors are of finite fractal dimension.

Synthesis of invariant relations in observation and identification problems

Volodymir Shcherbak, *Slov'yansk, Ukraine*

The problem of a mathematical model unknown components (state vector, parameters) determination by output data for nonlinear dynamical systems is considered. A new method – the synthesis of invariant relations is proposed. The method consists in the expansion of the original dynamical system by equations of its controlled prototype. Control in the additional system is used for the synthesis of some relations that are invariant for the extended system. These relations are dependent on the known output, phase vector of virtual model and unknown components of the initial system and must be synthesized in such way that corresponding invariant manifold becomes attracting for all trajectories of extended system. Then such relations may be considered in observation and identification problems as additional equations for unknown components of the mathematical model. The scheme of the construction of invariant relations is used for synchronization problems, as well

as for the stabilization ones. The application of this approach to the observation and identification problems for mechanical systems – a rigid body, a system of rigid bodies, a chain of coupled nonlinear oscillators are considered. Further the scheme of the construction of invariant relations is developed for synchronization and stabilization problems too.

Optimal damping coefficient of rotating Timoshenko beams

Jarosław Woźniak, *Szczecin, Poland*

Stability analysis of complex mechanical systems described by partial differential equations is one of important problems of mathematical control theory. Here we focus on the model of Timoshenko beams slowly rotating in a horizontal plane. One end of the beam is clamped to a motor disk, controlling the rotation, second end moves freely. We introduce a viscoelastic damping effect with respect to a parameter taking into account additional rotation of the cross-section area of the beam. In the regular case of the beam (i.e. for values $\gamma > 1$ of the physical parameter of the beam material) we find that the spectrum of the system consists of two families, one of which is located on the imaginary axis; thus we obtain lack of stability of the system.

In the special case, $\gamma = 1$, we find two families of points of spectrum of the system, both located on the left-hand side of imaginary axis. We also find that real parts of the right-most family change with the changes of damping coefficient ν, and maximal value of real parts of eigenvalues of the system is -0.0332416 obtained for $\nu = 2,54189$. The disk-beam system with this specific damping coefficient dissipates the energy the fastest possible way.

Open second order systems and model representations of operators

Vladimir Zolotarev, *Kharkiv, Ukraine*

We study open system

$$\begin{cases} \ddot{h}(t) + B\dot{h}(t) + Ah(t) = \varphi^* \sigma u(t); \\ h(0) = h_0;\ \dot{h}(0) = h_1,\ t \in [0, T]; \\ v(t) = u - i\varphi h, \end{cases}$$

where B, A are linear bounded operators in a Hilbert space H, σ is a self-adjoint operator in a Hilbert space E; $\varphi :\ H \to E$, and $h(t) \in H$; $u(t), v(t) \in E$. In the case of $B = B^*$, explicit model realizations of the operators B and A are given in functional spaces, and the roots of the quadratic operator sheaf $\lambda^2 I + \lambda B + A$ are found.

Time-varying stabilization of nonlinear controllable systems by using fast oscillating feedback laws

Alexander Zuyev, *Magdeburg, Germany*

The goal of this lecture is to present a survey of stabilizability conditions for nonlinear controllable systems governed by ordinary differential equations. It is a well-known fact that any controllable linear system is stabilizable by a static feedback law. However, the question "Whether it is possible to stabilize an arbitrary controllable system by a state feedback?" has stimulated a number of non-trivial results for essentially nonlinear systems. These results are underlined in the introductory part of this lecture, where Brockett's, Coron's, and Dayawansa's necessary conditions are presented based on the degree theory. The main part of this lecture is devoted to the stabilization problem for nonlinear control-affine systems by means of a time-varying feedback control. It is assumed that the vector fields of the system satisfy Hormander's condition. A family of trigonometric open-loop controls is constructed to approximate the gradient flow associated with a Lyapunov function. These controls are applied for the derivation of a time-varying feedback law under the sampling strategy. By using Lyapunov's direct method, it is proved that the controller proposed ensures exponential stability of the equilibrium for the case of fast oscillating feedback. Several examples where such control design procedure is applied for the stabilization of nonholonomic mechanical systems are presented.

SHORT COMMUNICATIONS

Stability and stabilizability of time-delay neutral type dynamical systems

Pavel Barkhayev, *Kharkiv, Ukraine*
Grigory Sklyar, *Szczecin, Poland*

Interest in time-delay differential equations and the corresponding infinite-dimensional dynamical systems stems from the fact that many applied problems from physics, mechanics, biology, economics and engineering can be described by such equations. The problems of stability and stabilizability of time-delay systems have been studied intensively recent years due to their great importance. The majority of works deals with exponential stability (stabilizability) and a number of results has been obtained for both systems of retarded type and systems of neutral type. However, for systems of neutral type there appears to exist an essentially different kind of stability: asymptotic non-exponential stability. This fact is a consequence of the special behavior of spectrum of neutral type systems. The property of non-exponential stability is rather “delicate” and its analysis requires a complicated technique.

We analyze non-exponential stability and stabilizability for a class of neutral type systems with distributed delays. We consider corresponding infinite-dimensional dynamical systems in a Hilbert space, and study the critical case when there is a sequence of eigenvalues with real parts converging to zero. In this case, the system cannot be exponentially stable, but may be asymptotically non-exponentially stable. To obtain the conditions of asymptotic non-exponential stability we combine two techniques: existence of a Riesz basis of invariant finite-dimensional subspaces and boundedness of the resolvent in some subspaces of a special decomposition of the state space. For unstable systems, the techniques introduced enables to give the conditions of regular strong stabilizability.

Synthesis of bounded controls for a class of nonlinear systems

Maxim Bebiya, *Kharkiv, Ukraine*

We address the problem of bounded controls synthesis for a class of nonlinear systems with uncontrollable first approximation. Namely, we consider the system

$$\begin{cases} \dot{x}_1 = u, \quad |u(x)| \leq d, \\ \dot{x}_i = c_{i-1}x_{i-1} + f_{i-1}(t,x,u), \quad i = 2,\dots,n-1, \\ \dot{x}_n = c_{n-1}x_{n-1}^{2k+1} + f_{n-1}(t,x,u), \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control, $d > 0$ is a given number, $k = \frac{p}{q}$ ($p > 0$ is an integer, $q > 0$ is an odd integer), c_i ($i = 1,\dots n-1$) are real numbers such that $\prod_{i=1}^{n-1} c_i \neq 0$, $f_i(t,x,u) \in C(\mathbb{R} \times \mathbb{R}^n \times \mathbb{R})$ ($i = 1,\dots,n-1$) are such that $f_i(t,0,0) = 0$ for all $t \geq 0$.

The stabilization problem for system (1) was solved in [1]. In the case when $f_i(t,x,u) = 0$ ($i = 1,\dots,n-1$) the class of bounded finite-time stabilizing controls was constructed in [2] using V.I. Korobov's controllability function method [3]. In the present work we develop this approach to construct the class of controls $u = u(x) \in C(\mathbb{R}^n \setminus \{0\})$ such that: (i) for every $x_0 \in \mathbb{R}^n$ there exists a number $T(x_0) < +\infty$ such that $\lim_{t\to T(x_0)} x(t,x_0) = 0$, where $x(t,x_0)$ is a solution of the closed-loop system (1) satisfying the condition $x(0,x_0) = x_0$;

(ii) $|u(x)| \leq d$ for all $x \in \mathbb{R}^n \setminus \{0\}$.

[1] Bebiya M. O. and Korobov V. I. On Stabilization Problem for Nonlinear Systems with Power Principal Part// Journal of Mathematical Physics, Analysis, Geometry, 2016, 12, No. 2, 113–133.

[2] Bebiya M. O., Global synthesis of bounded controls for systems with power nonlinearity// Visnyk of V.N. Karazin Kharkiv National University, Ser. Mathematics, Applied Mathematics and Mechanics, 2015, 81, 36–51.

[3] Korobov V. I., The method of controllability function, R&C Dynamics, M.-Izhevsk, 2007 (in Russian).

The Solving of One Time-Optimal Problem on the Basis of the Markov Moment Min-problem with Even Gaps

Anna Bugaevskaya, *Belgorod, Russian Federation*

Consider the linear time-optimal problem

$$\dot{x} = Ax + bu, \quad |u| \le 1, \quad x \in E_n,$$

$$x(0) = x^0, \quad x(\Theta) = 0, \quad \Theta \to \min, \tag{1}$$

$$\operatorname{rank}(b, Ab, A^2b, \dots, A^{n-1}b) = n,$$

where A is a matrix of dimension $n \times n$, b is a n-dimensional vector, Θ is a time of motion from the point x^0 to the origin. Consider the solving of the time-optimal problem (1) for the case when the spectrum of the matrix A has the form $\sigma(A) = \{(2k-1)\lambda\}_{k=1}^{n}$. In the case of such matrix time-optimal problem (1) is reduced to the power Markov moment *min*-problem with even gaps [1]. The new generating function is suggested for finding the optimal time. The explicit form of the polynomial for which the set of nonnegative roots coincides with the set of switchings of the time-optimal control is given.

[1] Korobov V. I., Sklyar G. M. The Markov moment problem on the minimally possible closed interval, Dokl. Akad. Nauk SSSR, 1989, 308:3, pp. 525-528.

The solution of the linear time-optimal control problem with multidimensional control

Vladimir Florinsky, *Belgorod, Russian Federation*

Consider the linear time-optimal problem

$$\dot{x} = Ax + \textstyle\sum_{i=1}^{r} b_i u_i, \quad |u_i| \le 1, \quad i = \overline{1,r} \quad x \in E_n,$$

$$x(0) = x, \quad x(\Theta) = 0, \quad \Theta \to \min, \tag{1}$$

$$\operatorname{rank}(b_i, Ab_i, A^2b_i, \dots, A^{n-1}b_i) = n, \quad i = \overline{1,r},$$

where A is a matrix of dimension $n \times n$, b_i is a n-dimensional vector, Θ is a time of motion from the point x to the origin.

On the based of the analytical method of the solution of the canonical time-optimal problem, performance by V. I. Korobov and G. M. Sklyar a numerical solution of linear time-optimal control problem with control of arbitrary dimension (1) is proposed. The proposed solution is based on using numerical method of solving linear time-optimal problem with two-dimensional control [2].

[1] Korobov V. I., Sklyar G. M. The Time optimality and the power moment problem // Math. sbornik. - 1987. - 134 (176), no. 2 (10), - pp. 186 - 206.

[2] Florinsky V. V. Solution of linear time-optimal problem with two-dimensional control// Belgorod State University Scientific Bulletin Mathematics and Physics, 2015, no.5 (202), pp. 89 - 95.

Fundamental solution of an implicit linear differential equation over an arbitrary ring

Sergey Gefter, *Kharkiv, Ukraine*
Anna Goncharuk, *Kharkiv, Ukraine*

In the work we consider the convolution operation in the ring $K((\frac{1}{x}))$ of Laurent formal series having the form $b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ldots + \frac{a_n}{x^n} + \ldots$ over an arbitrary unitary ring K, which is not necessarily commutative. This operation is an algebraic analogue for the Hurwitz product of Laurent series, which is widely used in the theory of functions. Using this convolution we obtain some formula of Cauchy type for the solution of the following implicit linear inhomogeneous differential equation

$$by' + R(x) = y, \tag{1}$$

where $b \in K$ and $R \in K[x, \frac{1}{x}]$. This formula allows us to consider the Euler series

$$\mathcal{E}_b(x) = \frac{1}{x} - \frac{1!b}{x^2} + \frac{2!b^2}{x^3} - \frac{3!b^3}{x^4} \ldots$$

as a fundamental solution of the differential equation (1) in $K((\frac{1}{x}))$ and in $K[x, \frac{1}{x}]$.

We also consider the equation $by' + f(x) = y$, where $b \in \mathbb{Z}$, $f \in \mathbb{Z}[[x]]$, and show that the Euler series $\mathcal{E}_b(x)$ is its fundamental solution as well. To this end, we define the convolution of an element from $\frac{1}{x}\mathbb{Z}[[x]]$ with an element from $\mathbb{Z}[[x]]$, using the p-adic topology on $\mathbb{Z}$.

Asymptotic properties of essentially nonlinear systems with even-order resonances

Victoria Grushkovskaya, *Stuttgart, Germany*

This talk is devoted to the study of the decay rate of trajectories to a nonlinear system of ordinary differential equations with neutral linear approximation. Namely, it is assumed that the characteristic equation of the system has roots with negative real parts and purely imaginary roots related via even-order resonances. By using the center manifold reduction and the normal forms method, we obtain a class of Lyapunov functions and asymptotic stability conditions of the trivial solution for the system considered. The main result of this research states that, unlike the exponential stability of linear systems, solutions of essentially nonlinear systems exhibit the power decay estimate. It is shown that the order of such estimate may vary for systems with a diagonalizable matrix of linear approximation and with a matrix containing a Jordan block. A spring-pendulum system with resonant frequencies is considered as an example.

Dynamics of complex inverter pendulum: stability and control with time-delayed feedback

Natalya Kizilova, *Kharkiv, Ukraine*
Michail Karpinski, *Kharkiv, Ukraine*

Body sway of healthy volunteers and patients with locomotory disorders at different postures is usually modeled as n-link inverted pendulum with a position feedback control. Those models perfectly describe the postural sway of healthy individuals in the sagittal plane (backward-upward), while for the more complex sway in the frontal plane (left-right) as well as for patients with body asymmetry, congenital pathology, trauma or diseases; more complex multi-pendulum models are needed. In the present study a brief thorough review of the elaborated and tested non-linear and linearized dynamical models of complex pendulums for different 2-leg and 1-leg stances is given. The comprehensive analysis of the results of experimental measurements on groups of healthy volunteers as well as patients of Kharkov Institute of Spine and Joints Pathology are presented. The control functions $\vec{u}(t)$ have been computed for different groups by substitution of the measured data into the corresponding mathematical models. It was shown, for the healthy volunteers the control functions can be presented as $\vec{u} = \vec{a} \cdot \vec{q} + \vec{b} \cdot \dot{\vec{q}}$, where $\vec{a}, \vec{b} = const$, $\vec{q}$ are the generalized coordinates of the pendulum, $\cdot\dot{\vec{q}}$ means the time derivative, while for the diseased and elderly patients the nonlinear functions with time delay feedback control have been found.

Robust stabilization of one class of nonlinear systems

Anatolii Lutsenko, *Kharkiv, Ukraine*

We consider the robust linear stabilization problem for a family of nonlinear controllable systems that contains functional-parametric uncertainties and depends on the control nonlinearly. We obtain sufficient conditions for robust stabilization and synthesize state linear controllers that perform robust stabilization. We also obtain necessary conditions for robust stabilization that are close to sufficient. The synthesis is based on the method of Lyapunov functions.

Operators with non-basis family of eigenvectors and the well-posedness of evolution equations

Vitalii Marchenko, *Kharkiv, Ukraine*

In a joint work with Prof. Dr. Grigory M. Sklyar we study the well-posedness of Cauchy problems for linear evolution equations with linear unbounded operators possessing the following properties. Operators have purely imaginary eigenvalues, which essentially cluster at the infinity, and corresponding eigenvectors are dense, minimal, but not uniformly minimal, hence do not form a Schauder basis. For this purpose we introduce some special Hilbert and Banach spaces, consider special classes of real sequences and apply the discrete Hardy inequality. We found conditions guaranteeing that the operator will be an infinitesimal generator of the strongly continuous group as well as conditions under which the operator will not generate even a strongly continuous semigroup in corresponding spaces. Thereby we obtain the results on the well/ill-posedness of the corresponding Cauchy problems. It turned out that the well-posedness of the equation essentially depends on the character of the asymptotic behavior of eigenvalues at the infinity. Our results complement the remarkable results of G.Q. Xu & S.P. Yung (2005) and H. Zwart (2010) on the Riesz basis property for spectral families. We also obtain explicit formulas of solutions and show that the constructed strongly continuous groups are unbounded but belong to the class of polynomially bounded groups.

Conditions of multigroup and multifield existence

Vladimir Mashtalir, *Kharkiv, Ukraine*
Vladislav Shlyakhov, *Kharkiv, Ukraine*

Developing truly intelligent methods for approximate reasoning under conditions of explicit and implicit information (which may be imprecise, uncertain, incomplete, conflicting, partially true) requires additional conceptual and algorithmic layer to the existing approaches which group elements together by indistinguishability, similarity, proximity or functionality in arbitrary feature or signal spaces. Indistinguishability and similarity, with their explication being equivalence and tolerance respectively, are often used as a basis for construction of systems designed for balancing insufficiency and redundancy. One of possible approaches is to use multialgebraic systems as mathematical apparatus for synthesis and analysis of internal, external and contextual properties of partitions and coverings, collective structure of a family of classes and hierarchical structure represent a possible foundation for qualitative/quantitative characterization of levels of abstraction, detail, control, explanation, difficulty, organization and so on. To generate multialgebraic systems of a known algebraic type (group, ring, field, etc.), it is important that ternary relations which induce these systems are defined on a Cartesian cube of some set, i.e. ternary relations should have a common carrier as an algebraic structure. From another hand, it means that a one-to-one correspondence takes place between equivalence classes formed upon comparison of one or another arbitrary ternary relation. It is typical for so-called difunctional relations. With this the connection between ternary and difunctional relations plays an important role in answering the question: under which conditions does ternary multirelation have a common carrier? Multialgebraic systems (first of all, conditions of multigroup and multifield existence) with Cartesian cube as a common carrier are studied in the paper.

Representation of convex Hamilton-Jacobi equations in optimal control theory

Arkadiusz Misztela, *Szczecin, Poland*

Existence and uniqueness of solutions to a Hamilton–Jacobi equation

$$\begin{aligned} -V_t + H(t,x,-V_x) = 0 \quad &\text{in} \quad (0,T)\times\mathbb{R}^n, \\ V(T,x) = g(x) \quad &\text{in} \quad \mathbb{R}^n, \end{aligned} \tag{1}$$

with H convex with respect to the last variable can be proved by associating H to an optimal control problem. It is possible, if there exists sufficiently regular triple (A, f, l) satisfying the equality

$$H(t,x,p) = \sup_{a\in A}\{\,\langle\, p\,, f(t,x,a)\,\rangle\; - \; l(t,x,a)\,\}. \tag{2}$$

Rampazzo [3] and Frankowska–Sedrakyan [1] prove that having sufficiently regular Hamiltonian H one can construct adequately regular triple (A, f, l) satisfying the equality (2). Rampazzo [3] called this triple a faithful representation of the equation (1) in optimal control theory.

We show a new method of construction of a faithful representation for a wider class of Hamiltonians than it has been obtained before. Actually, we get two types of representations: with a compact and a noncompact control set A depending on regularity of the Hamiltonian. We also present the stability of representations.

[1] Frankowska H., Sedrakyan H., Stable representation of convex Hamiltonians, Nonlinear Anal., 2014, 100, 30–42.

[2] Misztela A., Representation of convex Hamilton-Jacobi equations in optimal control theory, arXiv:1507.01424

[3] Rampazzo F., Faithful representations for convex Hamilton–Jacobi equations, SIAM J. Control Optim., 2005, 44(3), 867–884.

On asymptotic growth of solutions of delay differential equations of neutral type

Piotr Polak, *Szczecin, Poland*
Grigory Sklyar, *Szczecin, Poland*

We consider the following class of delay differential equations of neutral type

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)d\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)d\theta,$$

where $z(t) \in \mathbb{C}^n$, A_{-1} is fixed, nondegenerate $n\times n$ matrix and A_2, A_3 are $n \times n$ matrices with entries in $L_2(-1, 0)$.

We study asymptotic behaviour of the solutions of above equations. The main approach is to interpret delay equations as linear ordinary differential equations

$$\dot{x} = \mathcal{A}x, \quad x \in M_2 = \mathbb{C}^n \times L_2$$

in Hilbert space M_2. There will be presented some growth estimations of the norm of the corresponding semigroup of operators and of individual solutions. Moreover the notion of polynomial stability will be presented and some criterion of polynomial stability of neutral systems will be derived in terms of location of the spectrum of the semigroup generator. As an application a control problem will be also considered, that is the asymptotic behaviour of diameters of reachable sets is shown.

Mathematical models of suspension flows through porous medium

Sergey Poslavskii, *Kharkiv, Ukraine*
Boris Kaplin, *Kharkiv, Ukraine*
Anton Kuznietsov, *Kharkiv, Ukraine*

Suspension flow through porous media is described by the system of differential equations with partial derivatives. This system consists of the continuity equation for fluid, Darcy's law and equation of kinetics of sedimentation/leaching of particles in colmatage/suffosion process. Also differential equations for concentrations of the dissolved species and/or balance equations for suspended solid particles should be included if needed. As porosity can vary in time, the change of permeability is determined using the Kozeny-Carman equation for example. The report discusses the possibility of colmatage/suffosion processes control by providing the relevant boundary conditions. Results of simulations are presented. In some cases solutions have the form of travelling waves (or close to this form). Weak discontinuous solutions are also allowed.

Interfacial stability of the ferrofluid in a constant and oscillating magnetic fields

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Applied mechanical problems require the study of new types of control, one of the most effective of which in ferrohydrodynamics is as stationary as well alternating magnetic field. Under the influence of external magnetic field the formation of ordered spatial configurations on the free surface of a magnetic fluid (MF) occurs as a significant example of self-organization processes in physical systems. The increase of magnetic field induction usually leads to the sequence of bifurcations of equilibrium states and, under certain conditions, to the emergence of free surface shapes, that have some kind of symmetry group. In this report, the initial stage of the free surface evolution of a ferrofluid is considered on the example of several physical systems [1],[2]. The loss of stability and the transition to a new equilibrium state of MF free surface in the supercritical magnetic field is studied. Results for parametric instability of MF equilibrium states under the influence of oscillating magnetic field and mechanical vibration are presented. Marginal stability boundaries are determined in the space of key physical parameters. It is shown that the range of values of this parameters is divided into zones, corresponding to the harmonic and subharmonic instabilities. The effect of viscosity and other physical parameters of the fluid on a threshold of parametric instability is studied.

[1] Borysov I. D., Potseluiev S. I., Yatsenko T. Yu. Instability of equilibrium and appearance of ordered spatial structures on the free surface of ferrofluid // Magnetohydrodynamics. – 2014. – Vol. 50, № 1. – pp. 3–12.

[2] Patsegon N. F., Potseluiev S. I. The stability of a free surface of viscous ferrofluid layer under the influence of alternating magnetic field and mechanical vibrations // Tekhicheskaya Mekhanika. – 2016. – № 2. - c. 71-84 (in Rus).

On robust feedback synthesis for the system of two coupled pendulums

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The paper deals with robust feedback synthesis for a mechanical system which consists of two pendulums coupled by a spring. The linearized equations of the motion of this pendulums are of the form:

$$\begin{cases} \ddot{\varphi}_1 = -\dfrac{m_1 g l_1 + kh^2}{m_1 l_1^2}\,\varphi_1 + \dfrac{kh^2}{m_1 l_1^2}\,\varphi_2 + u_1, \\ \ddot{\varphi}_2 = \dfrac{kh^2}{m_2 l_2^2}\,\varphi_1 - \dfrac{m_2 g l_2 + kh^2}{m_2 l_2^2}\,\varphi_2 + u_2, \end{cases}$$

where l_1 and l_2 lengthes of pendulums, m_1 and m_2 theirs masses, k is the spring stiffness. The lengthes from the suspension points of two pendulums to the spring attachment points are considered to be equal to each other and we denote them by h. We assume that pairs of forces u_1 and u_2 satisfy the inequality $\|(u_1, u_2)^*\| = \sqrt{u_1^2 + u_2^2} \leq 1$.

Our approach is based on the controllability function method conceived by V. I. Korobov in 1979 [1]. Suppose that the values of m_1, m_2, l_1, l_2 and h are known. Suppose that the spring stiffness k is unknown. For any value of k we find such control which steers an arbitrary initial point x_0 from some neighborhood of the origin to the origin in finite time $T(x_0, k)$. Besides this control is independent of k.

[1] Korobov V. I. The method of controllability function (Russian), R&C Dynamics, M.-Izhevsk, 2007: 1-576.

Generalized mathematical formulations for nuclear power plants lifetime management problems

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Nuclear power plants lifetime management problems are considered from the standpoint of mathematical control theory. Generalized mathematical formulations of the problem of impact of operational control on lifetime of critical components of the reactor nuclear power plants are proposed as a related system of initial-boundary problems. A nuclear power plant technological process control programme is considered as a control in proposed generalized mathematical formulations. Lifetime of elements of nuclear power plants is determined using the concept of damage, taking into account the impact of control programme to external influencing factors. It is shown that the semi-discretization of the related initial value problems of the generalized mathematical formulation will lead to the Cauchy problem for ordinary differential equations and the known problem of mathematical control theory.

Spectral assignment of infinitesimal operators corresponding to equations of neutral type

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We consider a class of the equations with delay with single term of neutral type:

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)d\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)d\theta,$$

where $z(t) \in \mathbb{C}^n$, A_{-1} is fixed, nondegenerate $n \times n$ matrix and A_2, A_3 are $n \times n$ matrices with entries in $L_2(-1,0)$.

Equation of this class can be interpreted [1] as a linear equation in Hilbert space with infinitesimal operator $\mathcal{A}$ that we consider as a perturbation of the operator $\tilde{\mathcal{A}}$ (for the case $A_2(\theta) = A_3(\theta) = 0$) by the also unbound operator Δ related by the distributed delay terms. The problem we study is to describe all possible perturbations of the spectral parameters of the operator $\mathcal{A}$ with respect to ones of $\mathcal{A}_0$. This problem was solved completely in [2, 3]. It is shown that almost all values of spectral parameters quadratically close to unperturbed ones can be achieved by the corresponding choice of the matrices $A_2(\cdot)$, $A_3(\cdot)$ from $L_2[-1; 0]$. The proof is based on the Riesz basis properties of families of exponentials.

Nonsmooth mapping of the trajectories of the control systems

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In 1987 V.I. Korobov and G.M. Sklyar gave the explicit analytic solution of the steering problem for the canonical control system. For this aim the new min-moment problem has been stated and solved. Their results become the basis for the following investigations in the linear control theory. In [1] the constructive method to solve the Bellman equation for the linear steering problem is proposed. The relation of the method with the min-moment problem has been stated. Another problem framed in the control theory is to obtain an exact analytic solution of the steering problem for an arbitrary linear control system. The constructive method to solve this problem is proposed in [2]. The approach is to find the mapping between optimal trajectories of a linear system and the optimal trajectories of the canonical one. In fact, the equivalence of the systems with the same qualitative behavior in a neighborhood of the stationary point is investigated. It turns out that such mapping is nonsmooth and this fact defines the substance of the method. The explicit form of this mapping was found for the linear control systems of the second order. The proposed approach we can apply to solve the synthesis problem for nonlinear control systems and for the linear systems with two-dimensional control [3].

[1] Smortsova T. I. Solution of the Bellmann equation and its connection with the moment problem// Visnyk Kharkiv. Univ., 2002, v. 542, pp. 3-12.

[2] Korobov V. I., Ivanova T. I.// Nonsmooth Mapping of Linear Control Systems, J. of Optim. Theory and Applic., 2001, v. 108, No 2, pp. 389-405.

[3] Smortsova T. I. Mapping of the linear systems of the second order with two-dimensional control//Visnyk Kharkiv. Univ., 2012, v. 1018, pp. 4-20.

AUTHOR INDEX

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